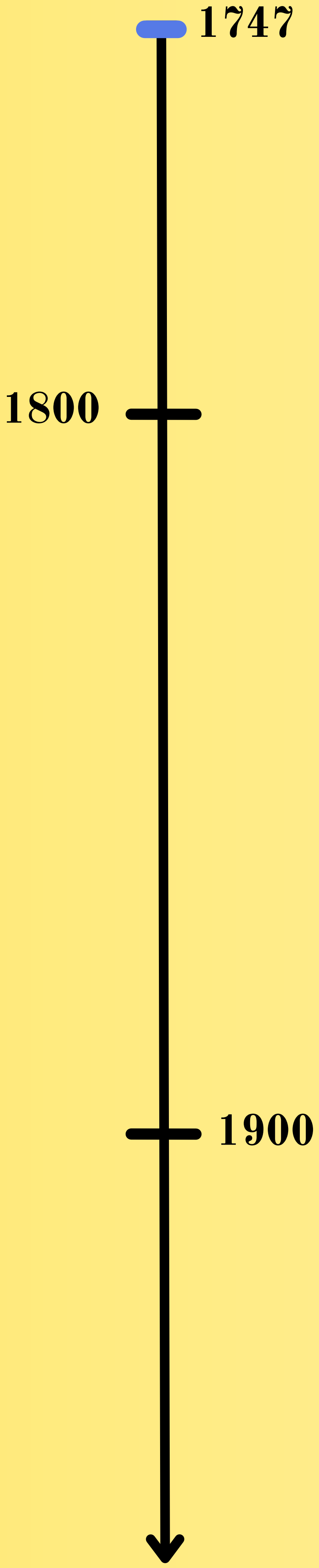
Fourier Analysis: The Catalyst of Modern Analysis

Samy LAHLOU and Nisrine SQALLI

Agenda

1. Early Stages of Fourier Analysis
2. Dirichlet's 1829 paper
3. Riemann's Integral and functions
4. Cantor's study of sets
5. From Lebesgue to now



Early Stages of Fourier Analysis

Jean Le Rond D'Alembert
[1717 -1783]

The wave equation (1747)

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$$

D'Alembert's solution to the wave equation

$$y = A(x - ct) + B(x + ct)$$

1747

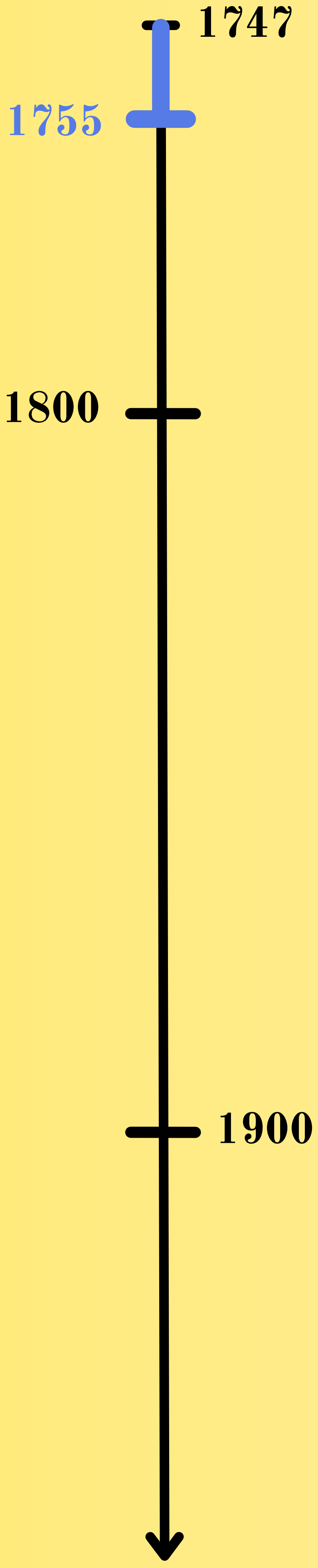
1800

1900

Early Stages of Fourier Analysis

Jean Le Rond D'Alembert
[1717 -1783]

Leonhard Euler
[1707 -1783]



Early Stages of Fourier Analysis

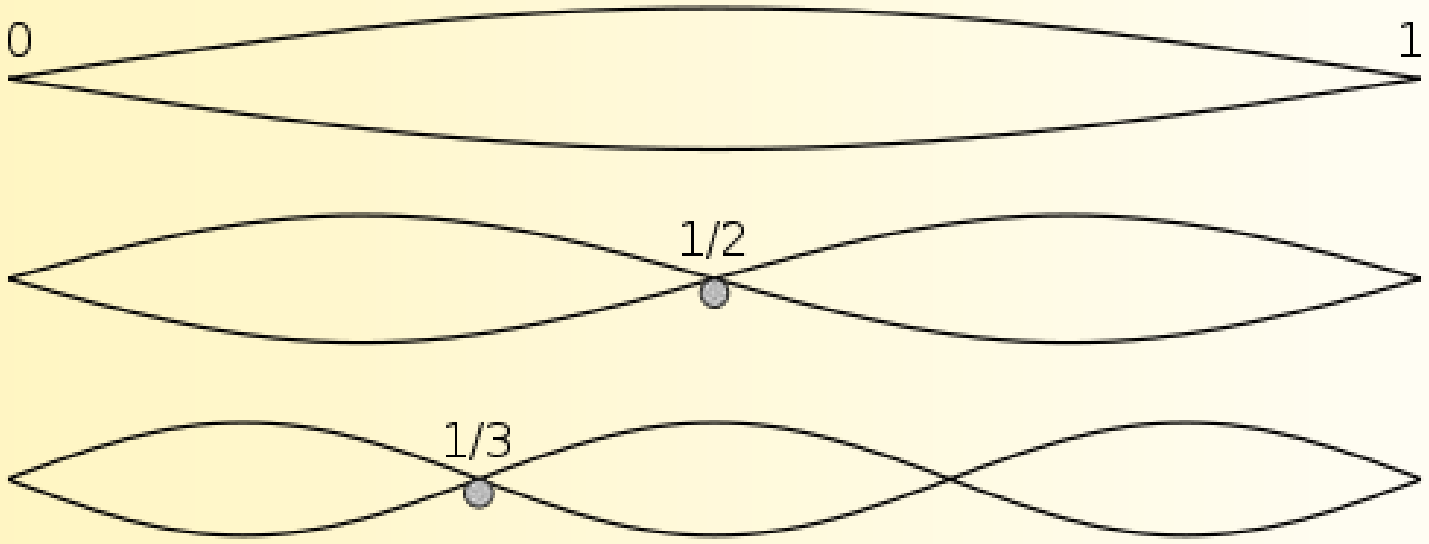
Daniel Bernoulli
[1700 -1782]

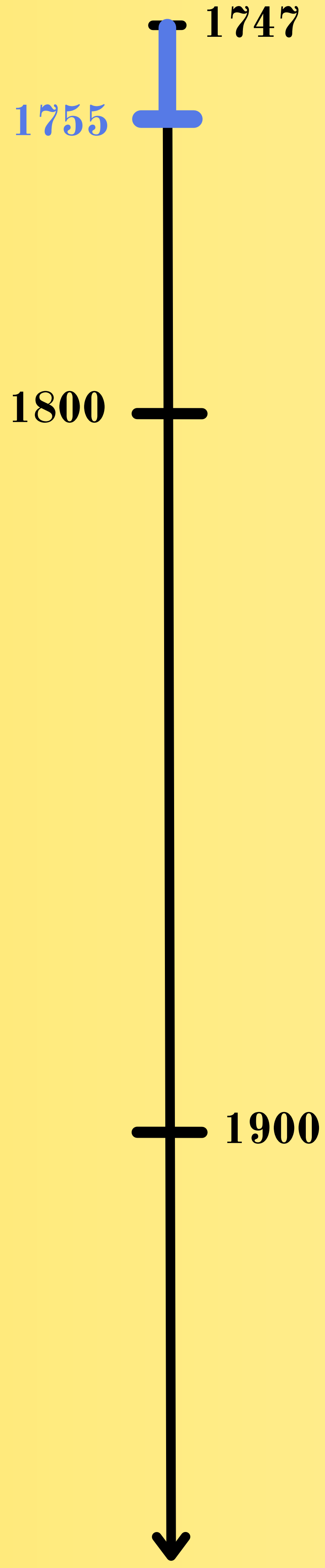
Bernoulli's solution to the wave equation

$$u(x,t) = \sum_{m=1}^{\infty} (A_m \cos(mt) + B_m \sin(mt)) \sin(mx)$$

Initial position

$$f(x) = \sum_{m=1}^{\infty} A_m \sin(mx)$$

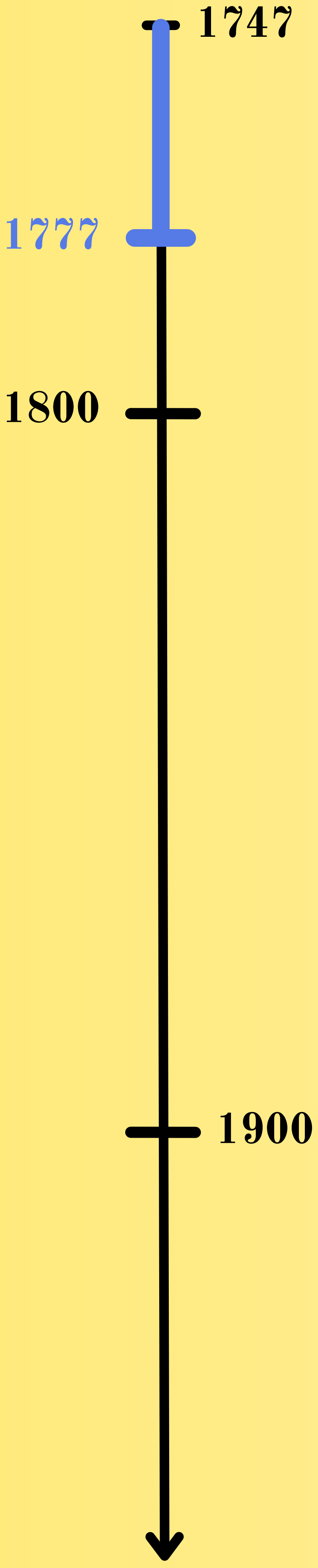




Early Stages of Fourier Analysis

Daniel Bernoulli
[1700 -1782]

Leonhard Euler
[1707 -1783]



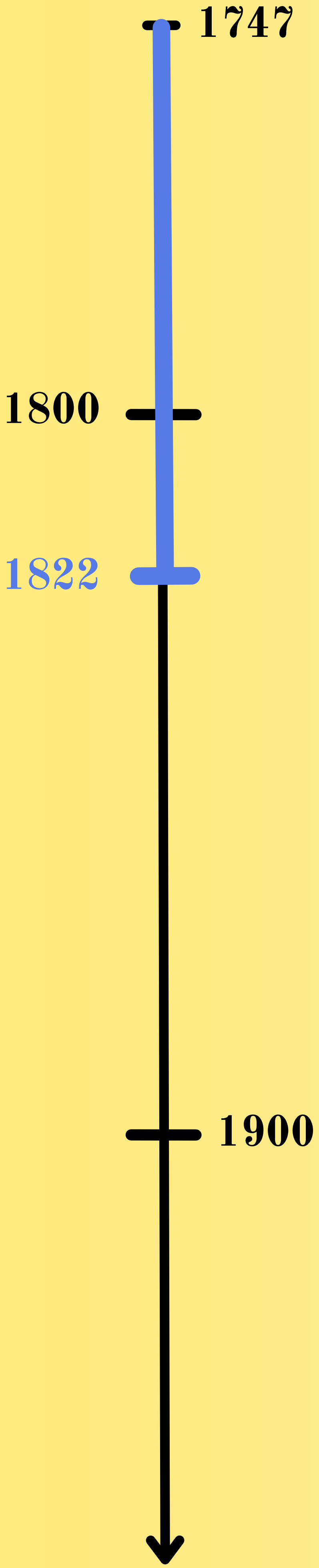
Early Stages of Fourier Analysis

Leonhard Euler
[1707 -1783]

Formula for the coefficients (1777)

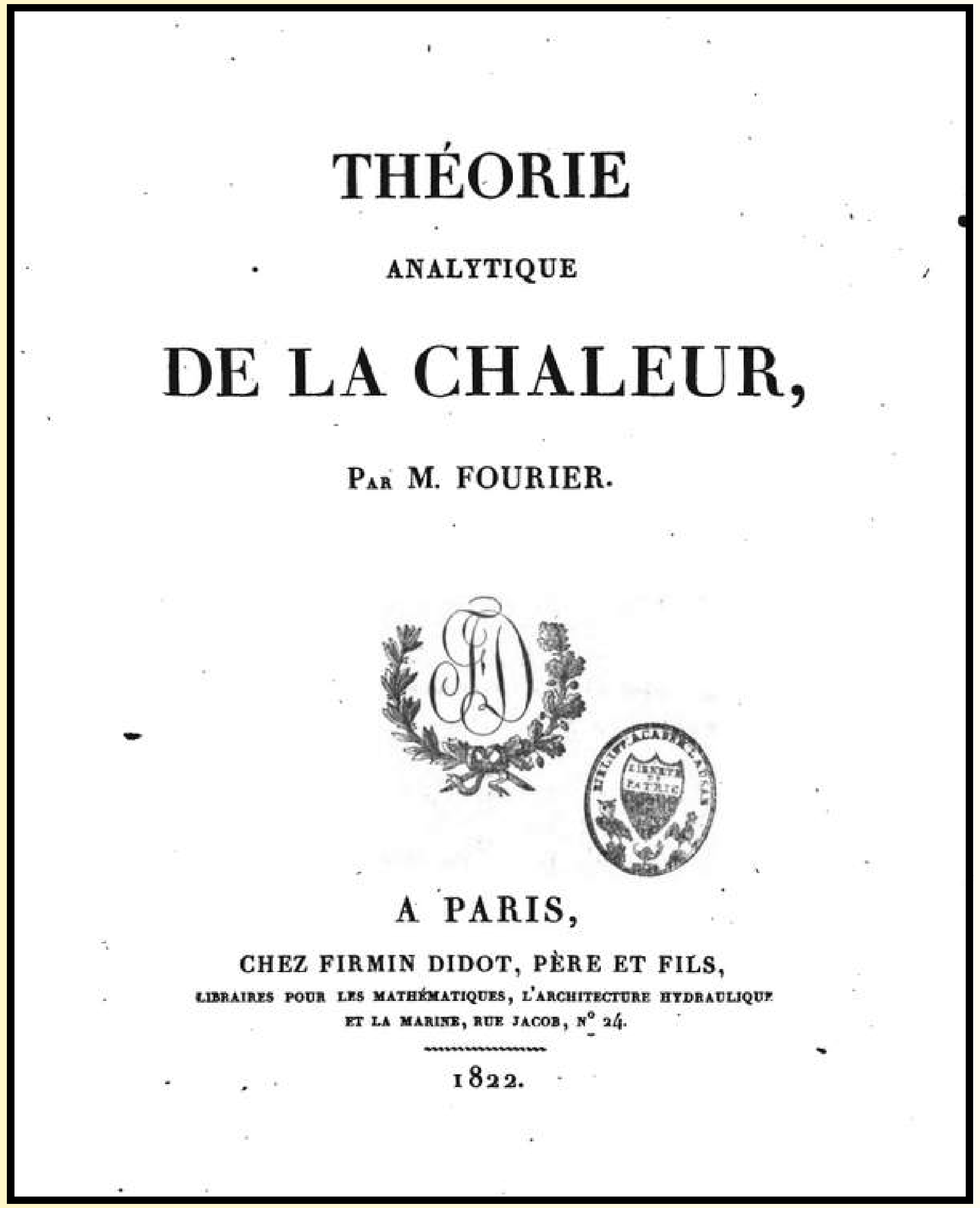
$$f(x) = \sum_{n=0}^{\infty} a_n \cos(nx)$$

$$a_n = \frac{2}{\pi} \int_0^{\pi} f(x) \cos(nx) dx$$



Early Stages of Fourier Analysis

Jean-Baptiste Joseph Fourier
[1768 -1830]



THÉORIE

ANALYTIQUE

DE LA CHALEUR,

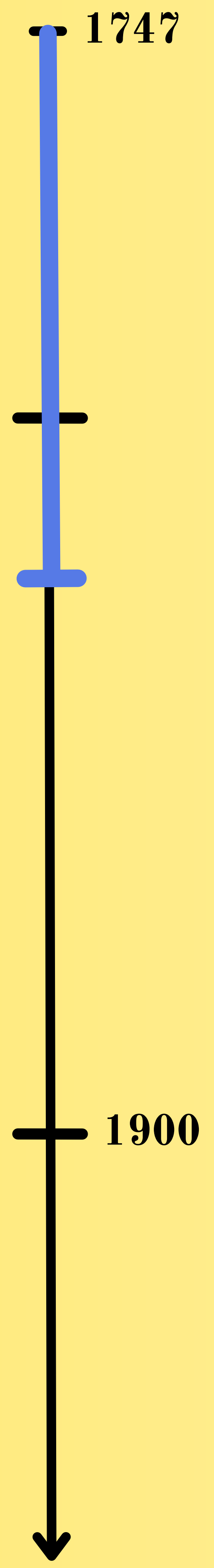
Par M. FOURIER.

A PARIS,

CHEZ FIRMIN DIDOT, PÈRE ET FILS,
LIBRAIRES POUR LES MATHÉMATIQUES, L'ARCHITECTURE HYDRAULIQUE
ET LA MARINE, RUE JACOB, N° 24.

1822.

"This theory will now form one of the most important branches of general physics."_preliminaries



Early Stages of Fourier Analysis

$$\varphi x = a \sin. x + b \sin. 2x + c \sin. 3x + d \sin. 4x + e \sin. 5x + \text{etc.}$$

En développant le second membre par rapport aux puissances de x, on aura les équations

$$\begin{aligned}
A &= a + 2\,b + 3\,c + 4\,d + 5\,e + \text{etc.}\\
B &= a + 2^3 b + 3^3 c + 4^3 d + 5^3 e + \text{etc.}\\
C &= a + 2^5 b + 3^5 c + 4^5 d + 5^5 e + \text{etc.}\\
D &= a + 2^7 b + 3^7 c + 4^7 d + 5^7 e + \text{etc.}\\
E &= a + 2^9 b + 3^9 c + 4^9 d + 5^9 e + \text{etc.} \qquad (a)\\
&\text{etc.}
\end{aligned}$$

La série $\sin. x = x - \frac{x^3}{2.3} + \frac{x^5}{2.3.4.5} + \frac{x^7}{2.3.4.5.6.7} + \text{etc.};$

nous fournira les quantités PQRST etc. En effet, la valeur du sinus étant exprimée par l'équation

$$\sin. x = x\left(1 - \frac{x^2}{1^2.\pi^2}\right)\left(1 - \frac{x^2}{2^2.\pi^2}\right)\left(1 - \frac{x^2}{3^2.\pi^2}\right)\left(1 - \frac{x^2}{4^2.\pi^2}\right)\left(1 - \frac{x^2}{5^2.\pi^2}\right) \text{etc.}$$

on aura $1 - \frac{x^2}{2.3} + \frac{x^4}{2.3.4.5} - \frac{x^6}{2.3.4.5.6.7} + \text{etc.}$

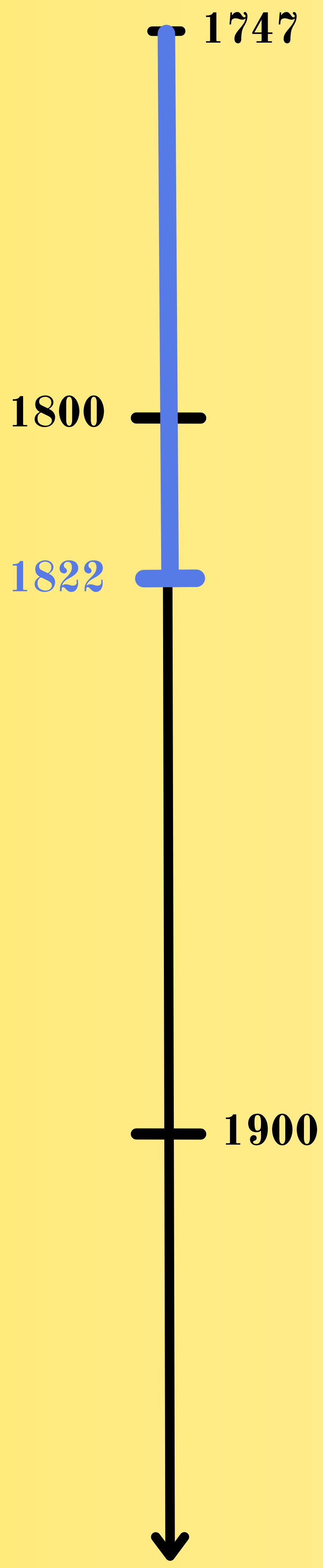
$$= \left(1 - \frac{x^2}{1^2.\pi^2}\right)\left(1 - \frac{x^2}{2^2.\pi^2}\right)\left(1 - \frac{x^2}{2^2.\pi^2}\right)\left(1 - \frac{x^2}{2^2.\pi^2}\right)\ldots \text{etc.},$$

$$\begin{aligned}
&a_1 = A_1, \quad a_2 + 2\,b_2 = A_2, \quad a_3 + 2\,b_3 + 3\,c_3 = A_3, \quad a_4 + 2\,b_4 + 3\,c_4 + 4\,d_4 = A_4\\
&a_2 + 2^3 b_2 = B_2, \quad a_3 + 2^3 b_3 + 3^3 c_3 = B_3, \quad a_4 + 2^3 b_4 + 3^3 c_4 + 4^3 d_4 = B_4\\
&a_3 + 2^5 b_3 + 3^5 c_3 = C_3, \quad a_4 + 2^5 b_4 + 3^5 c_4 + 4^5 d_4 = C_4\\
&a_4 + 2^7 b_4 + 3^7 c_4 + 4^7 d_4 = D_4\\
&a_5 + 2\,b_5 + 3\,c_5 + 4\,d_5 + 5\,e_5 = A_5\\
&a_5 + 2^3 b_5 + 3^3 c_5 + 4^3 d_5 + 5^3 e_5 = B_5\\
&a_5 + 2^5 b_5 + 3^5 c_5 + 4^5 d_5 + 5^5 e_5 = C_5\\
&a_5 + 2^7 b_5 + 3^7 c_5 + 4^7 d_5 + 5^7 e_5 = D_5\\
&a_5 + 2^9 b_5 + 3^9 c_5 + 4^9 d_5 + 5^9 e_5 = E_5\\
&\text{etc.} \qquad (b)
\end{aligned}$$

$$\begin{aligned}
\tfrac{1}{2}\pi\varphi x = \sin. x &\Big\{\varphi' o + \varphi''' o\left(\tfrac{\pi^2}{2.3} - \tfrac{1}{1^2}\right) + \varphi^{v} o\left(\tfrac{\pi^4}{2.3.4.5} - \tfrac{1}{1^2}\cdot\tfrac{\pi^2}{2.3} + \tfrac{1}{1^4}\right)\\
&+ \varphi^{vii} o\left(\tfrac{\pi^6}{2.3.4.5.6.7} - \tfrac{1}{1^2}\cdot\tfrac{\pi^4}{2.3.4.5} + \tfrac{1}{1^4}\cdot\tfrac{\pi^2}{2.3} - \tfrac{1}{1^6}\right) + \text{etc.}\Big\}\\
- \tfrac{1}{2}\sin. 2x &\Big\{\varphi' o + \varphi''' o\left(\tfrac{\pi^2}{2.3} - \tfrac{1}{2^2}\right) + \varphi^{v} o\left(\tfrac{\pi^4}{2.3.4.5} - \tfrac{1}{2^2}\cdot\tfrac{\pi^2}{2.3} + \tfrac{1}{2^4}\right)\\
&+ \varphi^{vii} o\left(\tfrac{\pi^6}{2.3.4.5.6.7} - \tfrac{1}{2^2}\cdot\tfrac{\pi^4}{2.3.4.5} + \tfrac{1}{2^4}\cdot\tfrac{\pi^2}{2.3} - \tfrac{1}{2^6}\right) + \text{etc.}\Big\}\\
+ \tfrac{1}{3}\sin. 3x &\Big\{\varphi' o + \varphi''' o\left(\tfrac{\pi^2}{2.3} - \tfrac{1}{3^2}\right) + \varphi^{v} o\left(\tfrac{\pi^4}{2.3.4.5} - \tfrac{1}{3^2}\cdot\tfrac{\pi^2}{2.3} + \tfrac{1}{3^4}\right)\\
&+ \varphi^{vii} o\left(\tfrac{\pi^6}{2.3.4.5.6.7} - \tfrac{1}{3^2}\cdot\tfrac{\pi^4}{2.3.4.5} + \tfrac{1}{3^4}\cdot\tfrac{\pi^2}{2.3} - \tfrac{1}{3^6}\right) + \text{etc.}\Big\}
\end{aligned}$$

(31 pages...)

Théorie Analytique de la Chaleur, Joseph Fourier, 1822



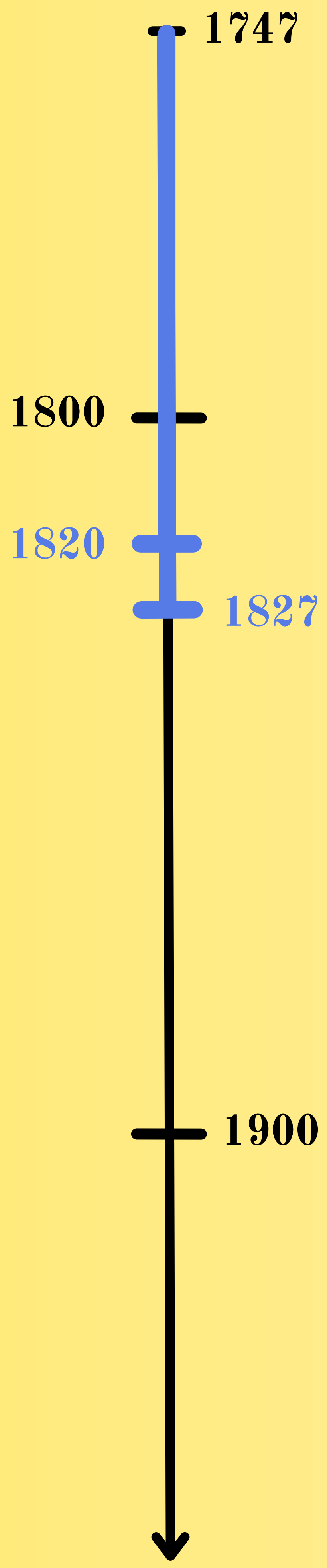
Early Stages of Fourier Analysis

$$\frac{1}{2}\pi\varphi x = \frac{1}{2}\int_0^\pi \varphi x\,dx + \cos.x\int_0^\pi \varphi x\cos.x\,dx$$
$$+ \cos.2x\int_0^\pi \varphi x\cos.2x\,dx + \cos.3x\int_0^\pi \varphi x\cos.3x\,dx + \text{etc.} \quad (n)$$

Fourier's Theorem

"This theorem and the previous one are suitable for all possible functions, whether we can express their nature by known means of analysis, or whether they correspond to curves drawn arbitrarily." _ page 241

Fourier's Theorem proof attempts

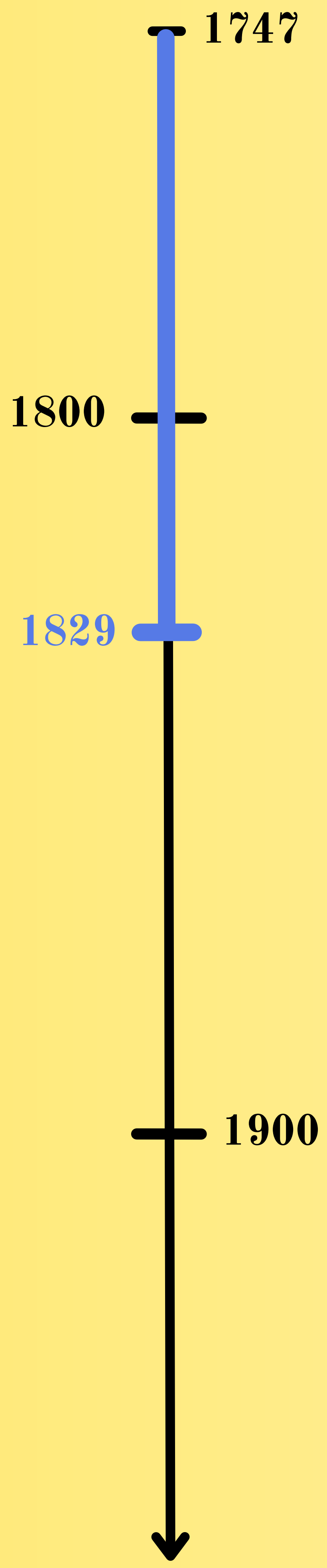


Siméon Denis Poisson
[1781 - 1840]

One proof attempt in 1820 but not rigorous enough

Augustin-Louis Cauchy
[1789 - 1857]

Two proof attempts (1826 & 1827) but not rigorous enough



Dirichlet's 1829 paper

Peter Gustav Lejeune Dirichlet
[1805 - 1859]

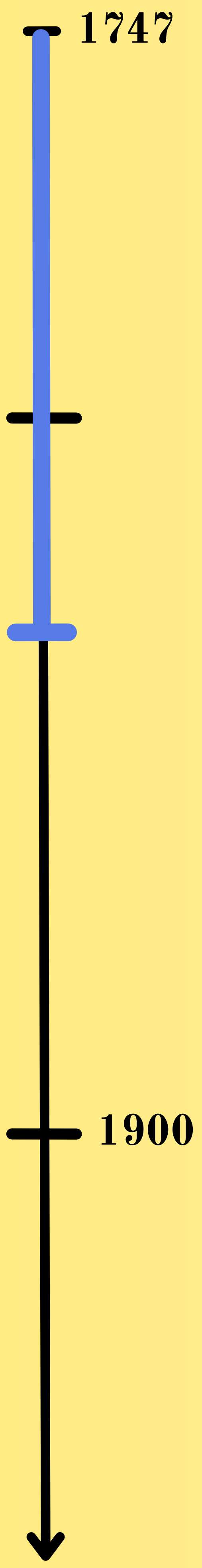
9.

Sur la convergence des séries trigonométriques qui servent à représenter une fonction arbitraire entre des limites-données.

(Par Mr. *Lejeune-Dirichlet*, prof. de mathém.)

On the convergence of trigonométric series that represents an arbitrary function between given limits.
(By Mr. Lejeune-Dirichlet, mathem. prof.)

January 1829



Dirichlet's 1829 paper

Cauchy's Limit Comparaison Test:

$$\lim_{n\to\infty} \frac{a_n}{b_n} = 1 \ \text{ and } \sum_{n=1}^{\infty} a_n < \infty$$

$$\implies \sum_{n=1}^{\infty} b_n < \infty$$

Modern Limit Comparaison Test:

$$a_n, b_n \geq 0, \ \lim_{n\to\infty} \frac{a_n}{b_n} = 1 \ \text{ and } \sum_{n=1}^{\infty} a_n < \infty$$

$$\implies \sum_{n=1}^{\infty} b_n < \infty$$

Cauchy's use of the LCT:

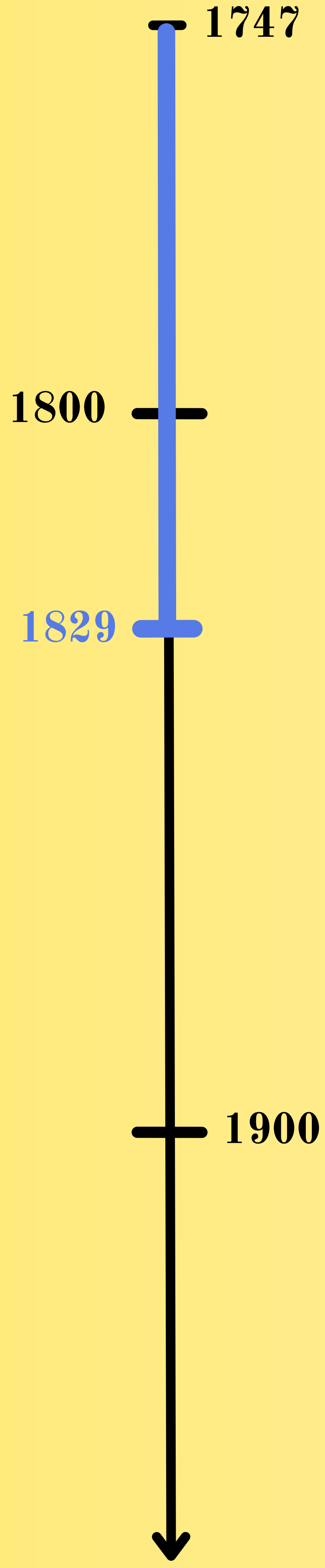
$$a_n = A_n \cos(nx) + B_n \sin(nx)$$

$$b_n = \frac{\sin(nx)}{n}$$

Dirichlet's counterexample:

$$a_n = \frac{(-1)^n}{\sqrt{n}} \left(1 + \frac{(-1)^n}{\sqrt{n}}\right)$$

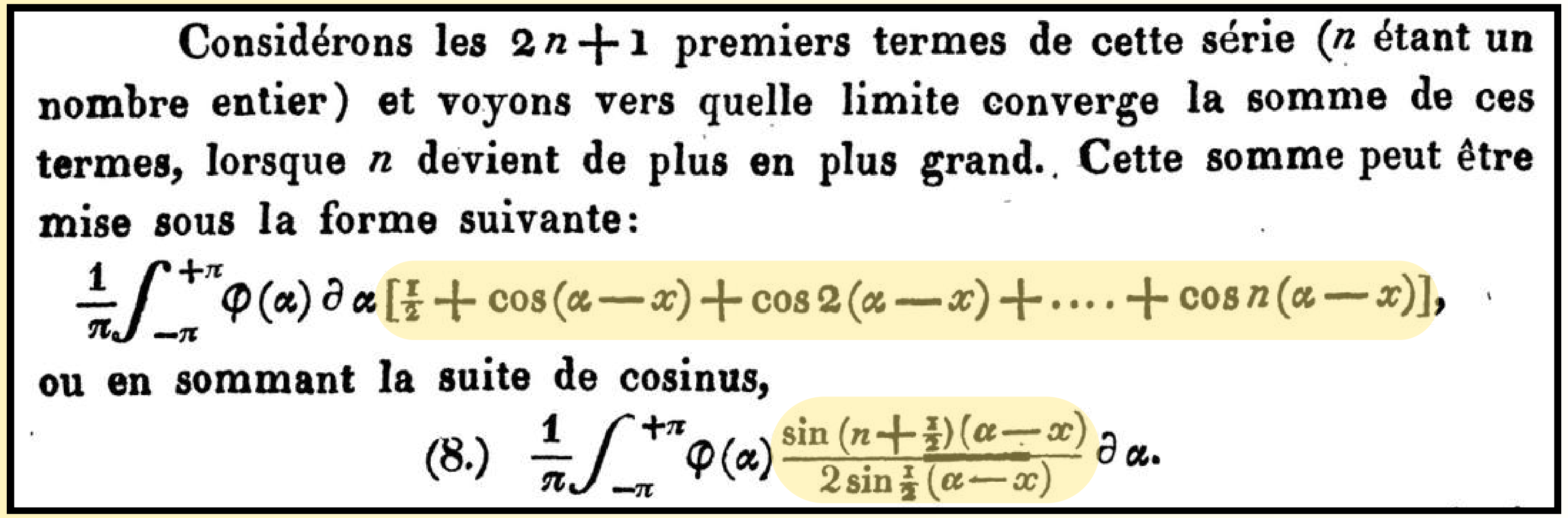
$$b_n = \frac{(-1)^n}{\sqrt{n}}$$



Dirichlet’s 1829 paper

Peter Gustav Lejeune Dirichlet
[1805 - 1859]

Used trigonometric identities to prove convergence (Dirichlet Kernel)



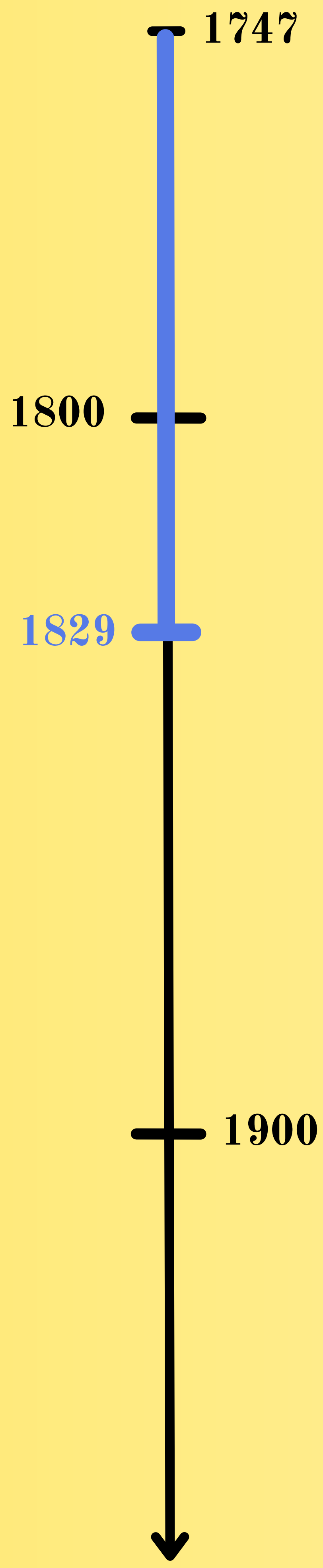
Considérons les $2n+1$ premiers termes de cette série (n étant un nombre entier) et voyons vers quelle limite converge la somme de ces termes, lorsque n devient de plus en plus grand. Cette somme peut être mise sous la forme suivante:

$$\frac{1}{\pi}\int_{-\pi}^{+\pi}\varphi(\alpha)\,\partial\alpha\left[\tfrac{1}{2}+\cos(\alpha-x)+\cos 2(\alpha-x)+\ldots+\cos n(\alpha-x)\right],$$

ou en sommant la suite de cosinus,

$$(8.)\quad \frac{1}{\pi}\int_{-\pi}^{+\pi}\varphi(\alpha)\frac{\sin(n+\frac{1}{2})(\alpha-x)}{2\sin\frac{1}{2}(\alpha-x)}\,\partial\alpha.$$

On the convergence of trigonométric series that represents an arbitrary function between given limits, page 166, Dirichlet, 1829



Dirichlet's 1829 paper

Peter Gustav Lejeune Dirichlet
[1805 - 1859]

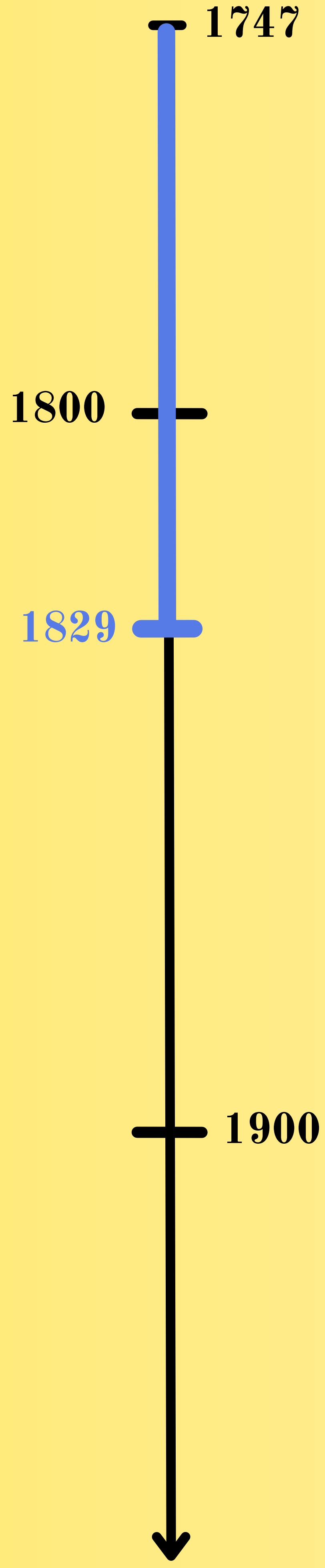
Dirichlet's Conditions

1° Can be Integrated

2° Doesn't have infinitely many maximas and minimas

3° If the functions yields a discontinuity, its value at the dicontinuity is the average between the values of the function on both sides of the discontinuity

Dirichlet's 1829 paper



Peter Gustav Lejeune Dirichlet
[1805 - 1859]

Dirichlet's Function :
A non-integrable function

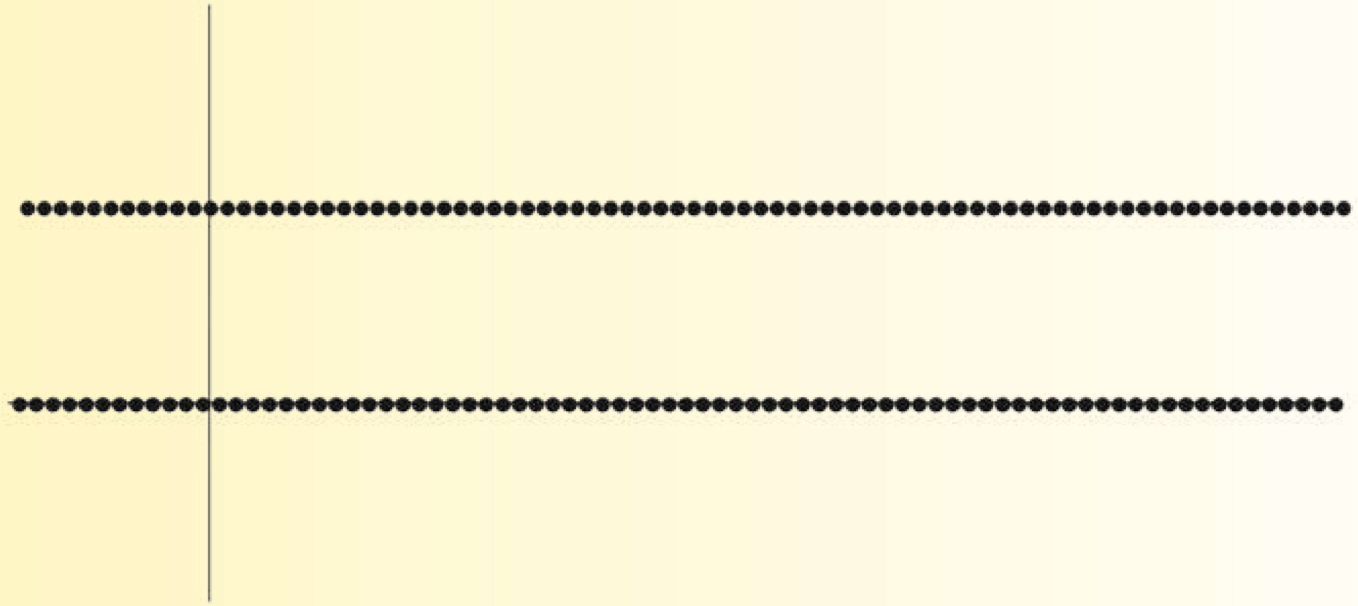
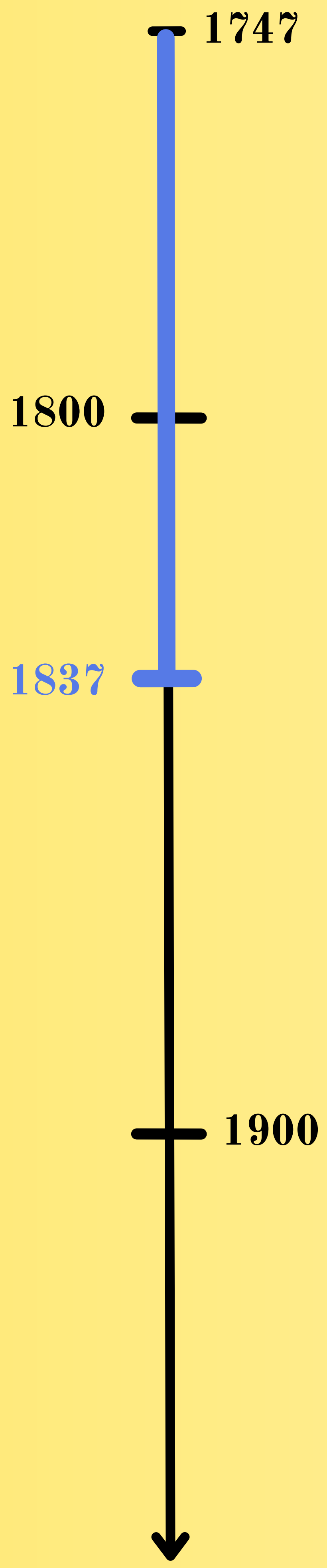


Figure taken from *Understanding Analysis* by Stephen Abbott

$$\varphi(x) = \begin{cases} c & \text{if } x \text{ is rational} \\ d & \text{if } x \text{ is irrational} \end{cases}$$



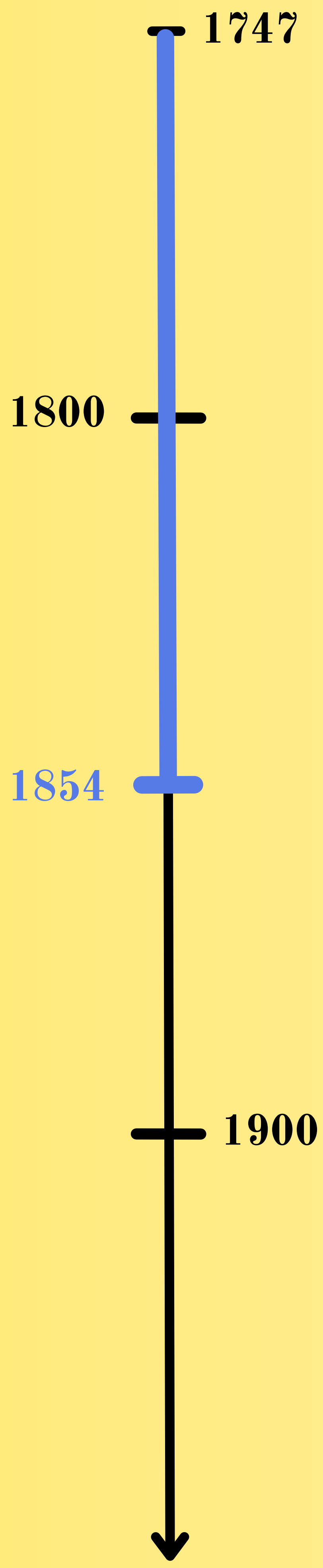
Dirichlet's 1829 paper

Peter Gustav Lejeune Dirichlet
[1805 - 1859]

Dirichlet's definition of Functions

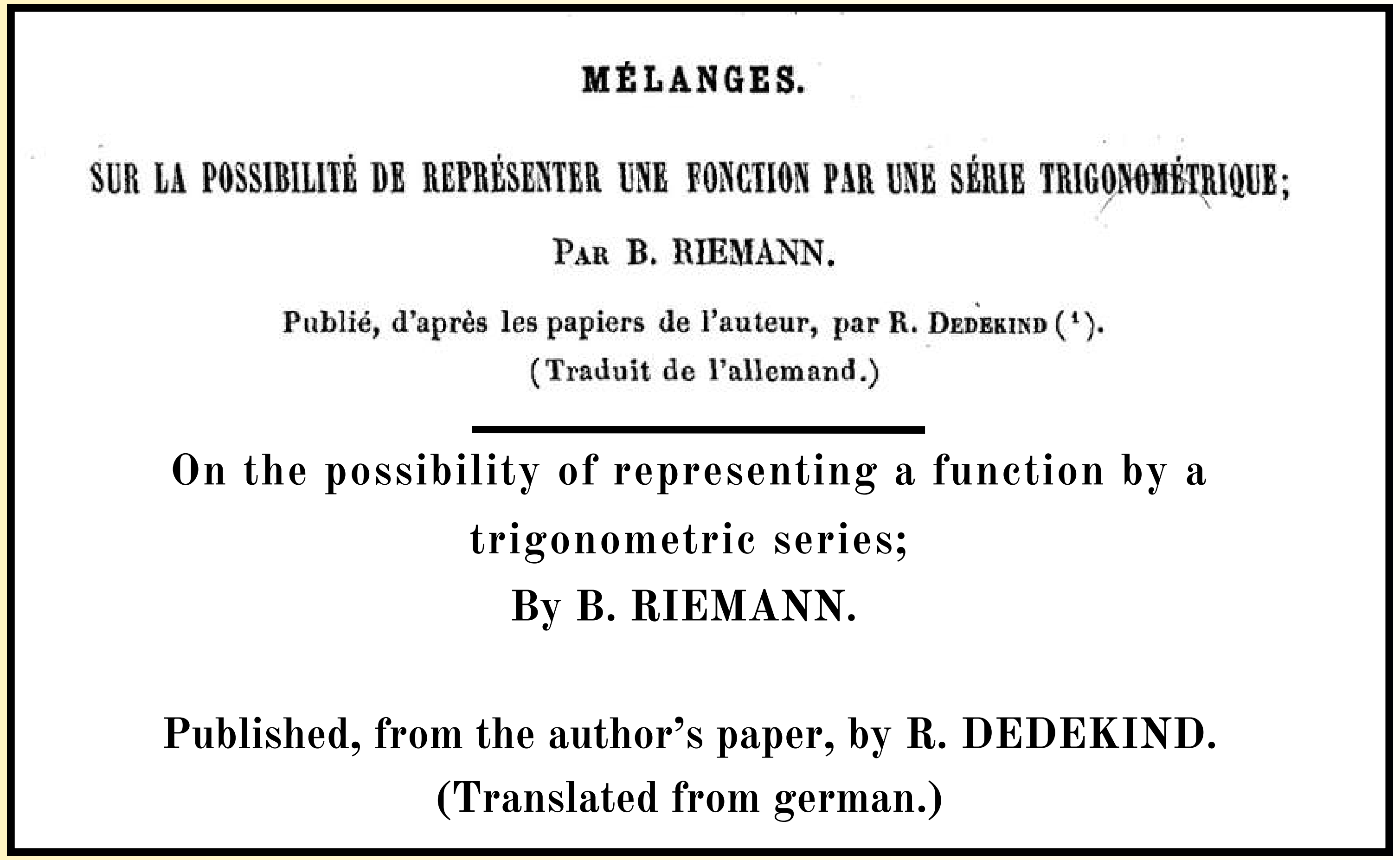
"It is not necessary that y be subject to the same rule as regards x throughout the interval, indeed one need not even be able to express the relation through mathematical operations"

– Dirichlet, 1837



Riemann's integral and functions

Bernhard Riemann
[1826 - 1866]



MÉLANGES.

SUR LA POSSIBILITÉ DE REPRÉSENTER UNE FONCTION PAR UNE SÉRIE TRIGONOMÉTRIQUE;

PAR B. RIEMANN.

Publié, d'après les papiers de l'auteur, par R. DEDEKIND (1).

(Traduit de l'allemand.)

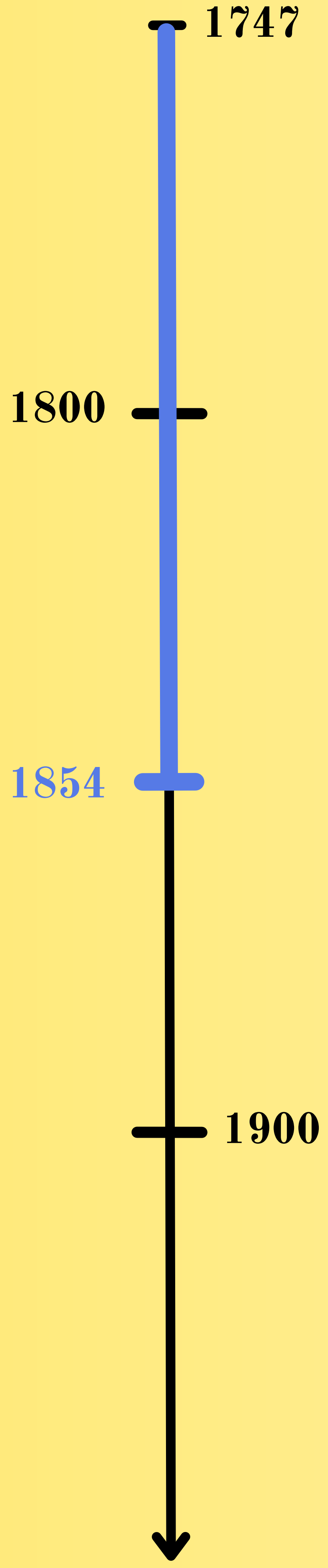
On the possibility of representing a function by a trigonometric series;

By B. RIEMANN.

Published, from the author's paper, by R. DEDEKIND.

(Translated from german.)

Written in 1854, published in 1867

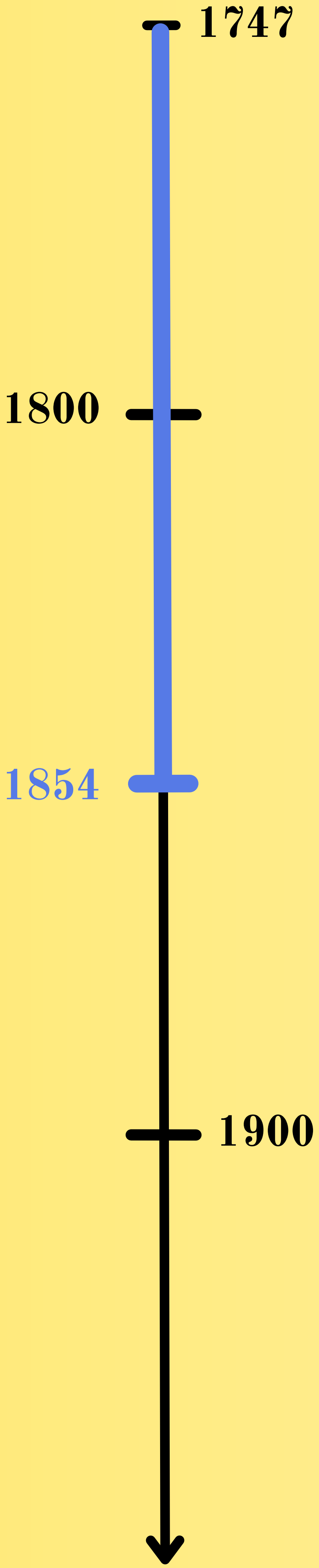


Riemann's integral and functions

Bernhard Riemann
[1826 - 1866]

"In fact, for all cases of nature, the only ones in question here, the question was completely resolved; because, [...] we can safely admit that the functions to which Dirichlet's research would not apply are not found in nature."

- Riemann, 1854



Riemann’s integral and functions

Bernhard Riemann
[1826 - 1866]

“In fact, for all cases of nature, the only ones in question here, the question was completely resolved; because, [...] we can safely admit that the functions to which Dirichlet's research would not apply are not found in nature.”

- Riemann, 1854

Motivations for Riemann’s work

1° Links to the principles of Infinitesimal Calculus

2° Applications to Number Theory

1747

1800

1854

1900

Riemann's integral and functions

Bernhard Riemann
[1826 - 1866]

Two classes of convergent series

Absolute convergence

Conditonal convergence

1747

1800

1854

1900

Riemann's integral and functions

Bernhard Riemann
[1826 - 1866]

Two classes of convergent series

Absolute convergence

Conditonal convergence

Riemann's Rearrangment Theorem

"It is clear now that the [conditionally convergent] series , by placing the terms in a suitable order, will be able to take any given value C [..].
It is only to series of the first class [that are absolutely convergent] that we can apply the laws of finite sums [...]."

- Riemann, 1854

1747

1800

1854

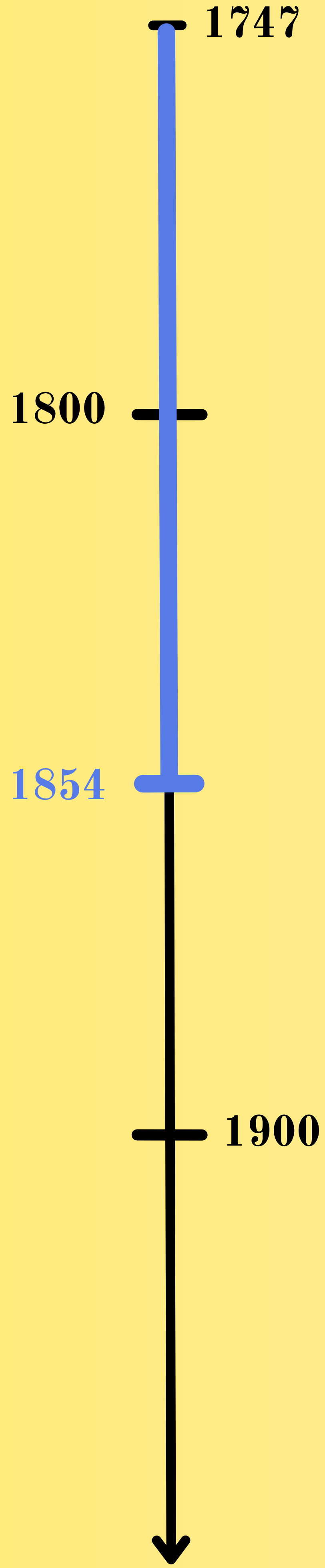
1900

Riemann's integral and functions

Bernhard Riemann
[1826 - 1866]

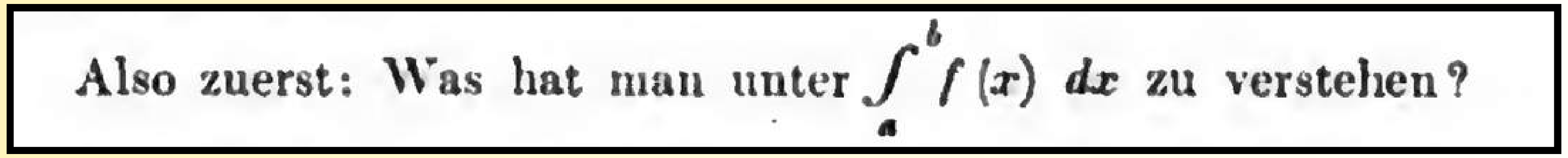
Also zuerst: Was hat man unter $\int_a^b f(x)\,dx$ zu verstehen?

"But first, what do we mean by $\int_a^b f(x)dx$ *?"_page 34*



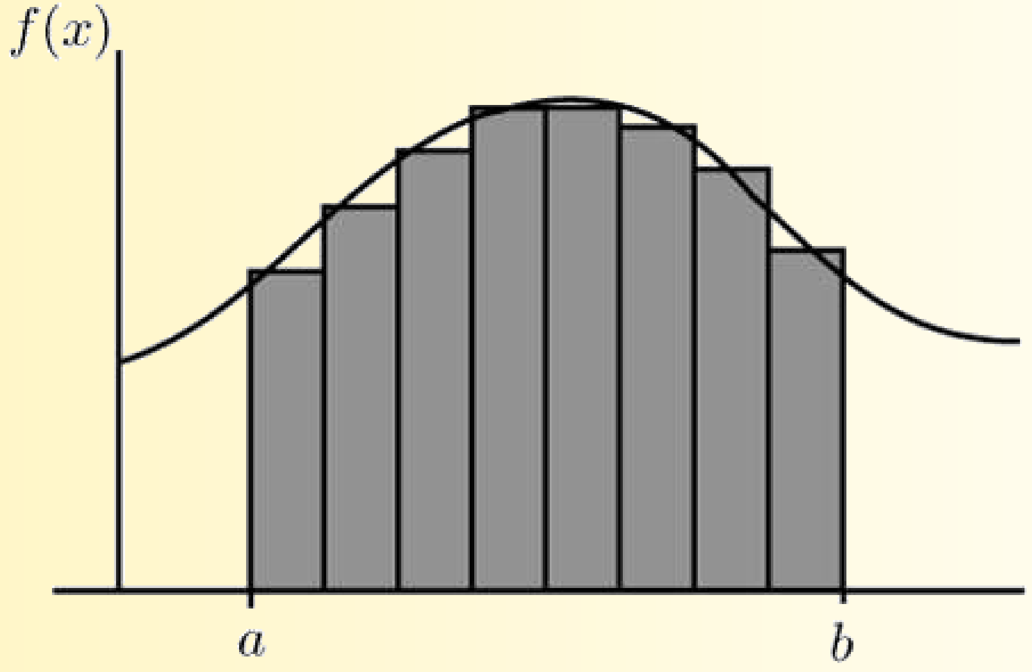
Riemann's integral and functions

Bernhard Riemann
[1826 - 1866]

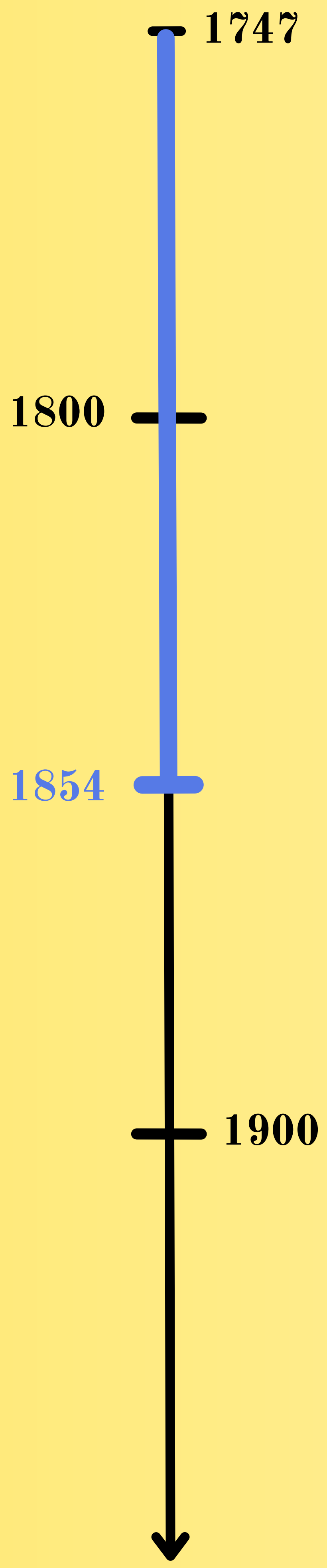


Also zuerst: Was hat man unter $\int_a^b f(x)\, dx$ zu verstehen?

"But first, what do we mean by $\int_a^b f(x)dx$ *?"_page 34*

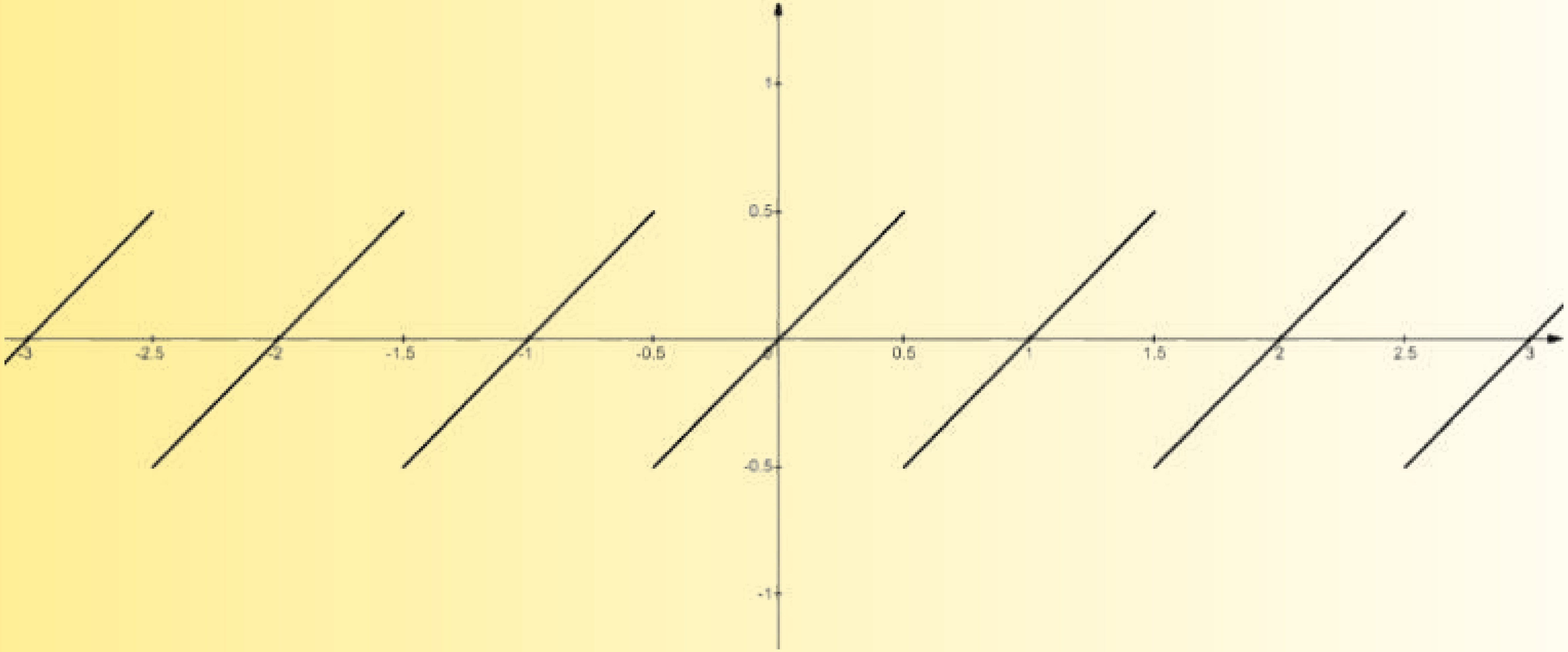


We can now integrate function with infinitaly many dicontinuities

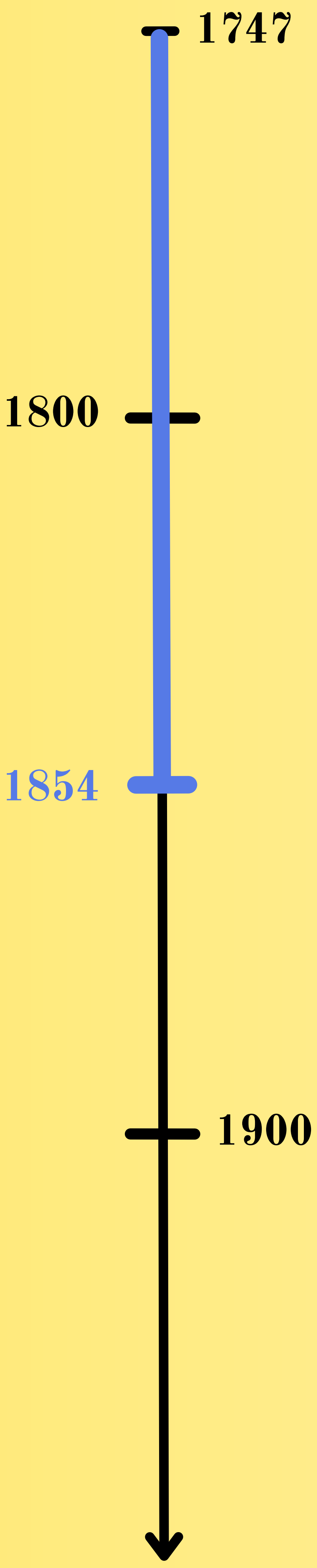


Riemann’s integral and functions

Riemann’s Pathological Function

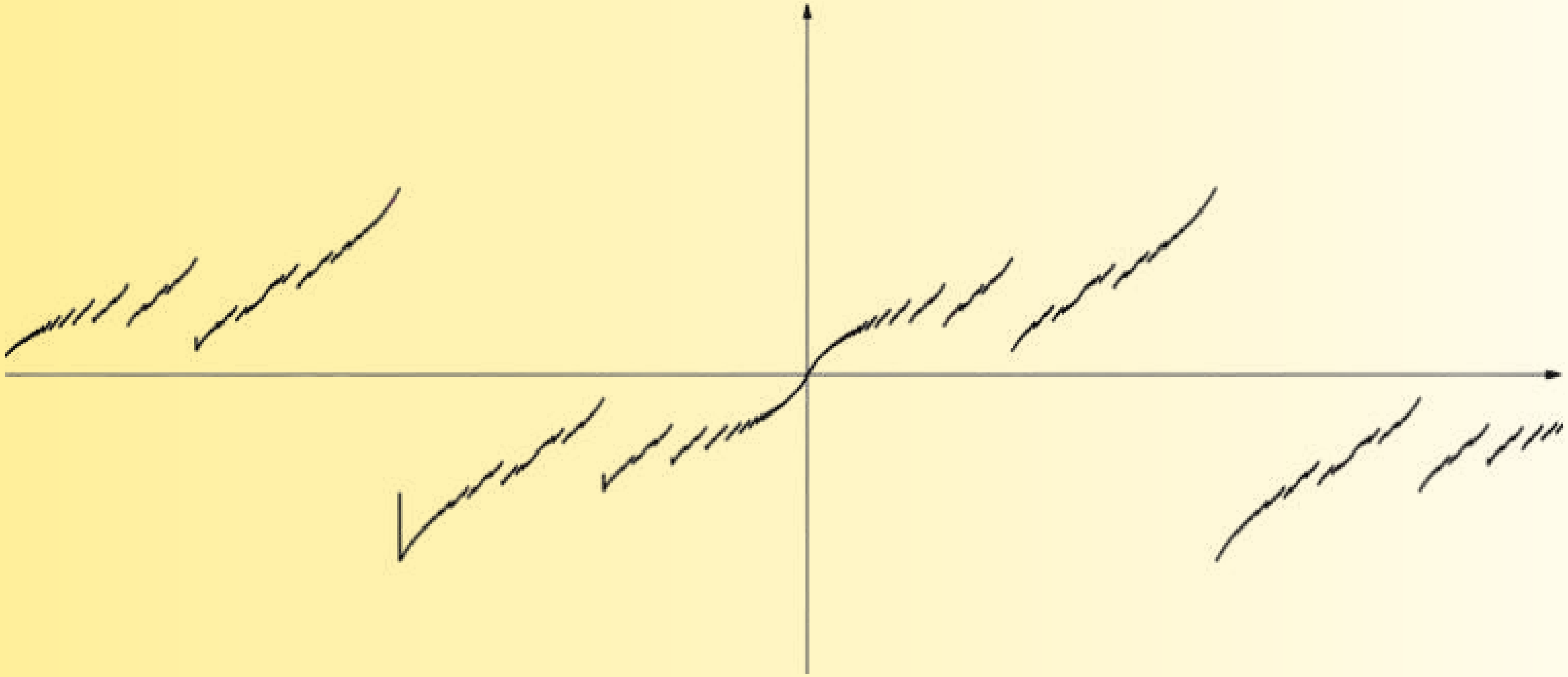


$$x \mapsto (x)$$

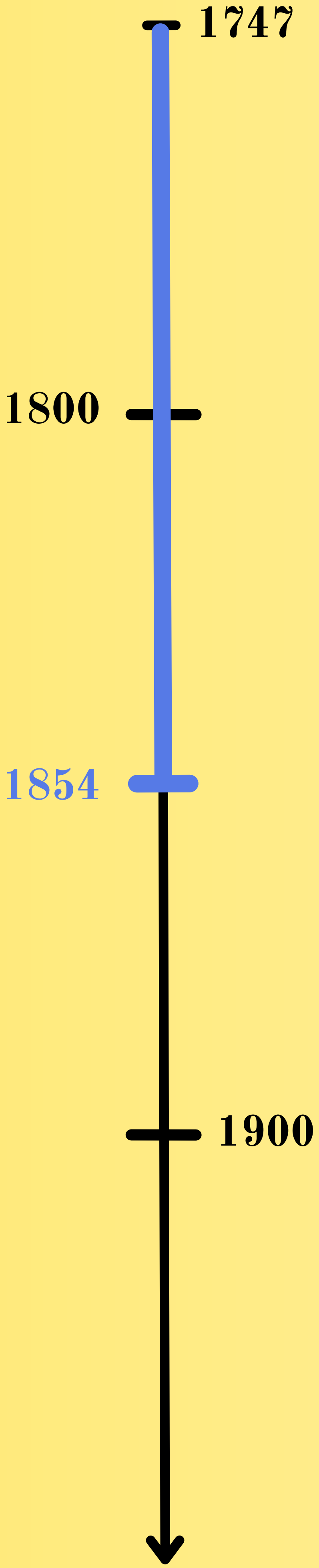


Riemann’s integral and functions

Riemann’s Pathological Function



$$f(x) = \frac{(x)}{1} + \frac{(2x)}{4} + \frac{(3x)}{9} + \ldots = \sum_{1}^{\infty} \frac{(nx)}{n^2}$$

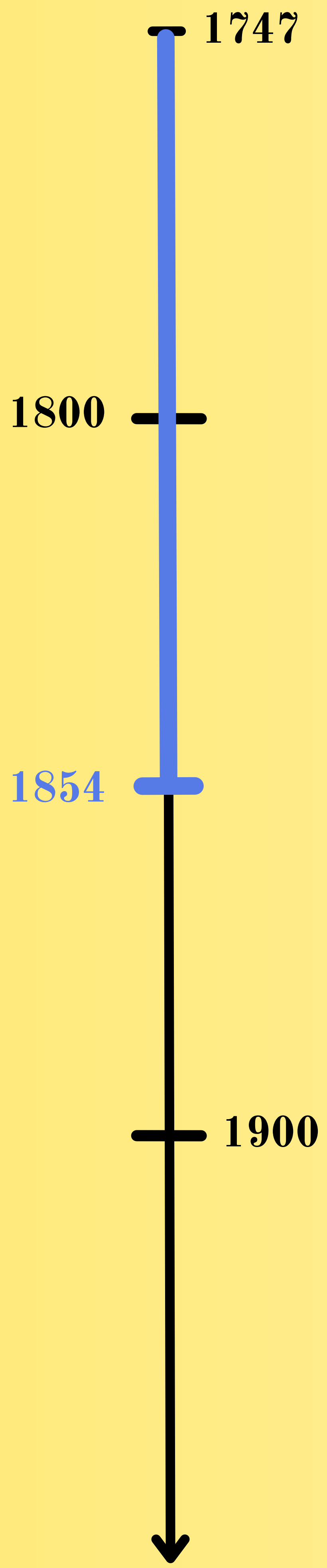


Riemann's integral and functions

Bernhard Riemann
[1826 - 1866]

If a function is integrable, it does not necessarily imply that it has finitely many maximas and minimas

$$1^\circ \not\Rightarrow 2^\circ$$



Riemann's integral and functions

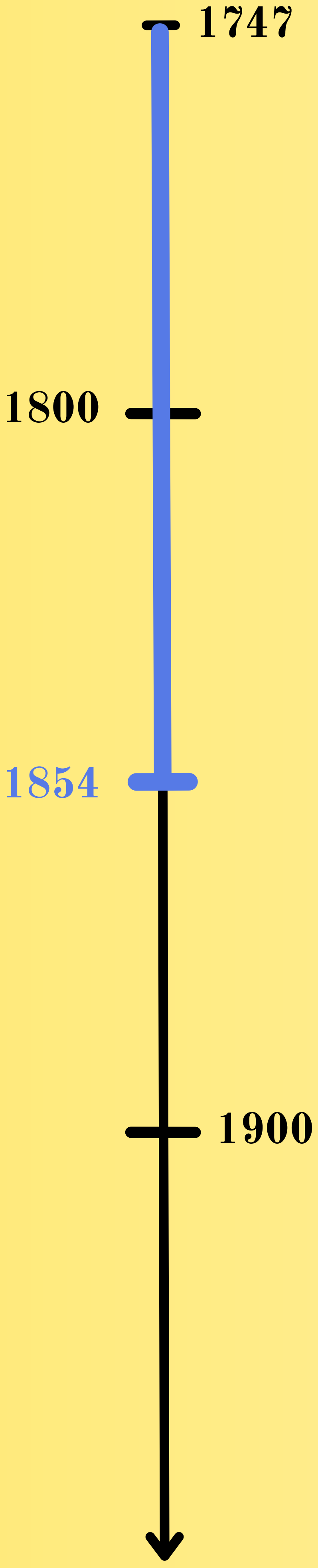
Bernhard Riemann
[1826 - 1866]

If a function is integrable, it does not necessarily imply that it has finitely many maximas and minimas

$$1^\circ \nRightarrow 2^\circ$$

If a function has finitely many maximas and minimas, it is integrable

$$2^\circ \Longrightarrow 1^\circ$$



Riemann's integral and functions

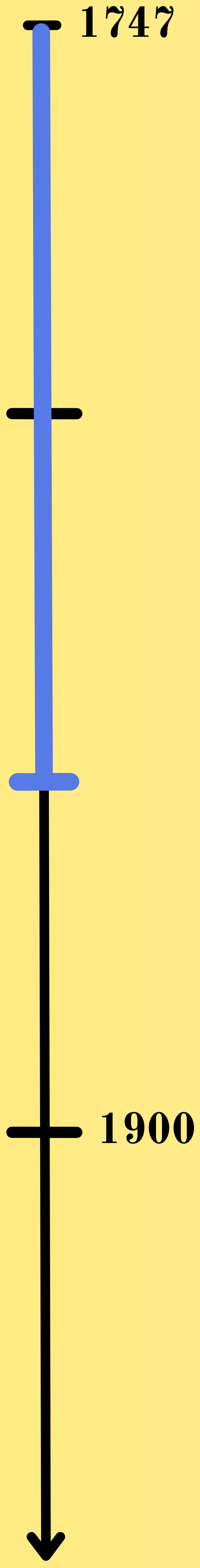
Bernhard Riemann
[1826 - 1866]

Riemann-Lebesgue Lemma

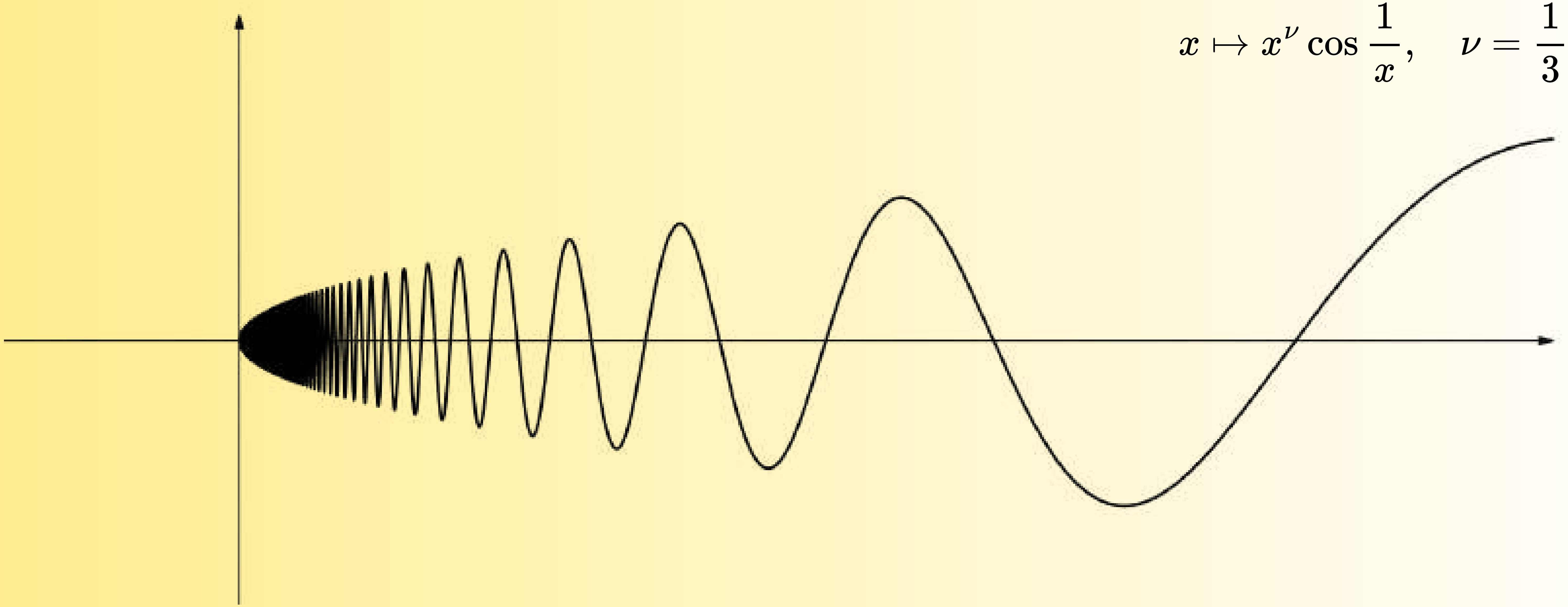
If *f(x)* is integrable (by Riemann's definition), then

$$\int_{-\pi}^{\pi} f(x)\sin(n(x-a))dx \to 0$$

as n goes to infinity and where a is a real number.



Riemann's integral and functions



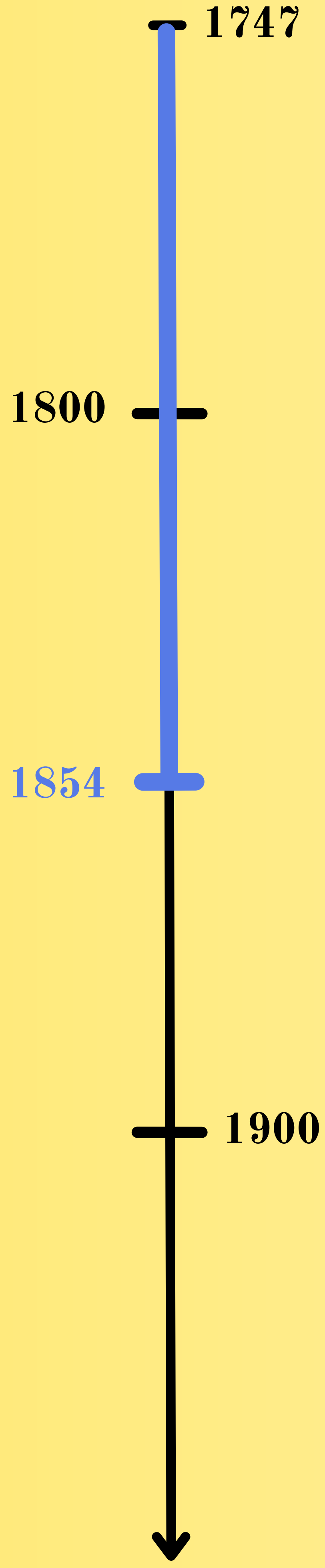
$$x \mapsto x^{\nu} \cos \frac{1}{x}, \quad \nu = \frac{1}{3}$$

$$f(x) = \frac{d(x^{\nu} \cos \frac{1}{x})}{dx}, \quad (0 < \nu < \frac{1}{2})$$

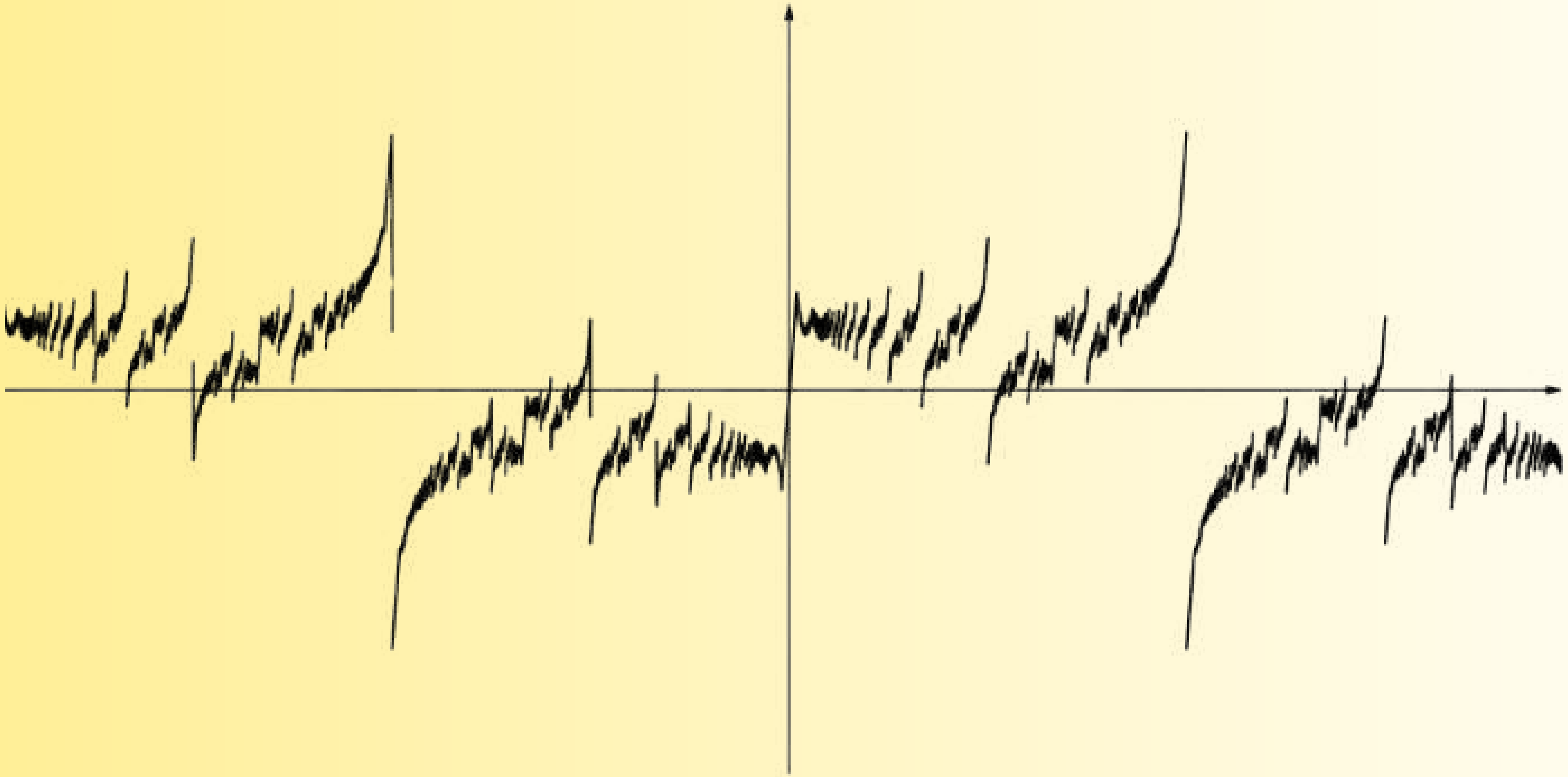
Infinitely many maximas and minimas

Integrable

Divergent Fourier Series



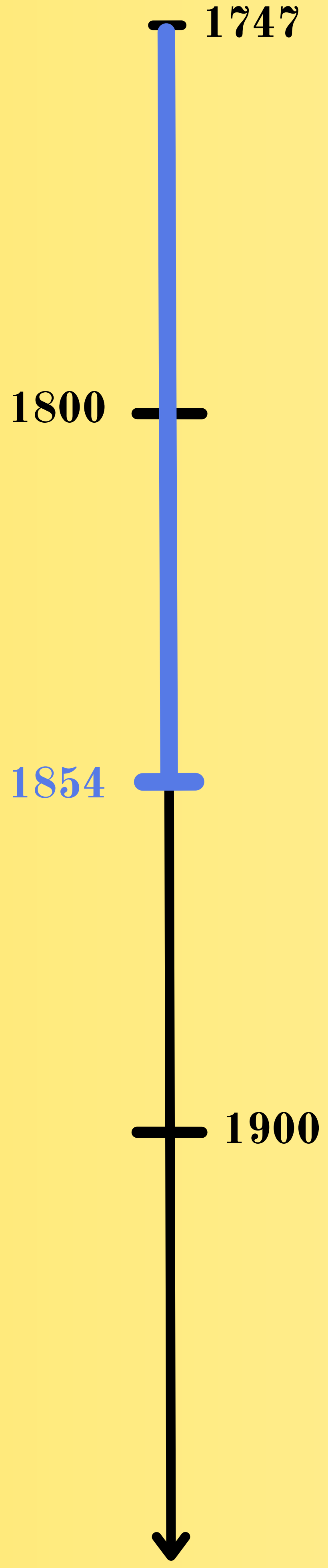
Riemann’s integral and functions



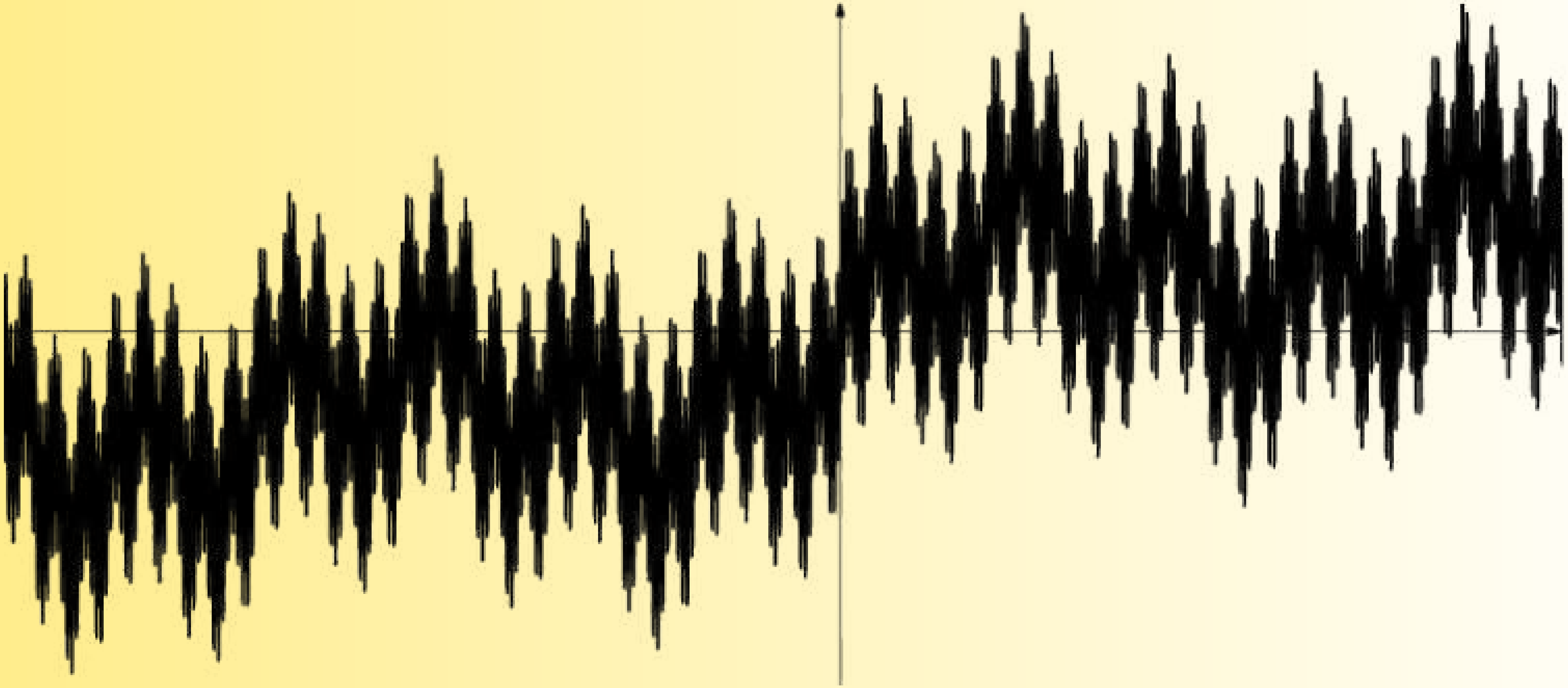
$$f(x) = \sum_{1}^{\infty} \frac{(nx)}{n}$$

Not Integrable

Convergent Fourier Series



Riemann’s integral and functions

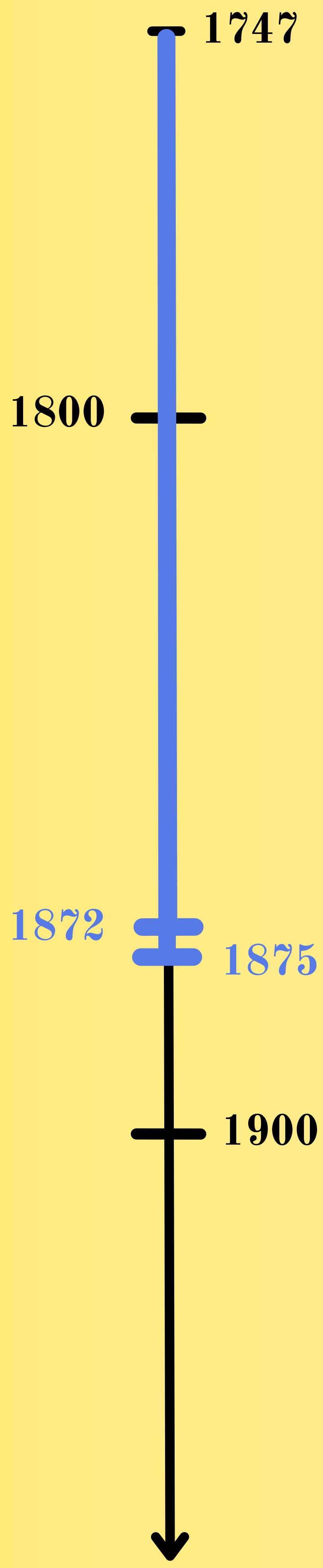


$$f(x) = \sum_{1}^{\infty} \sin((n!)x\pi)$$

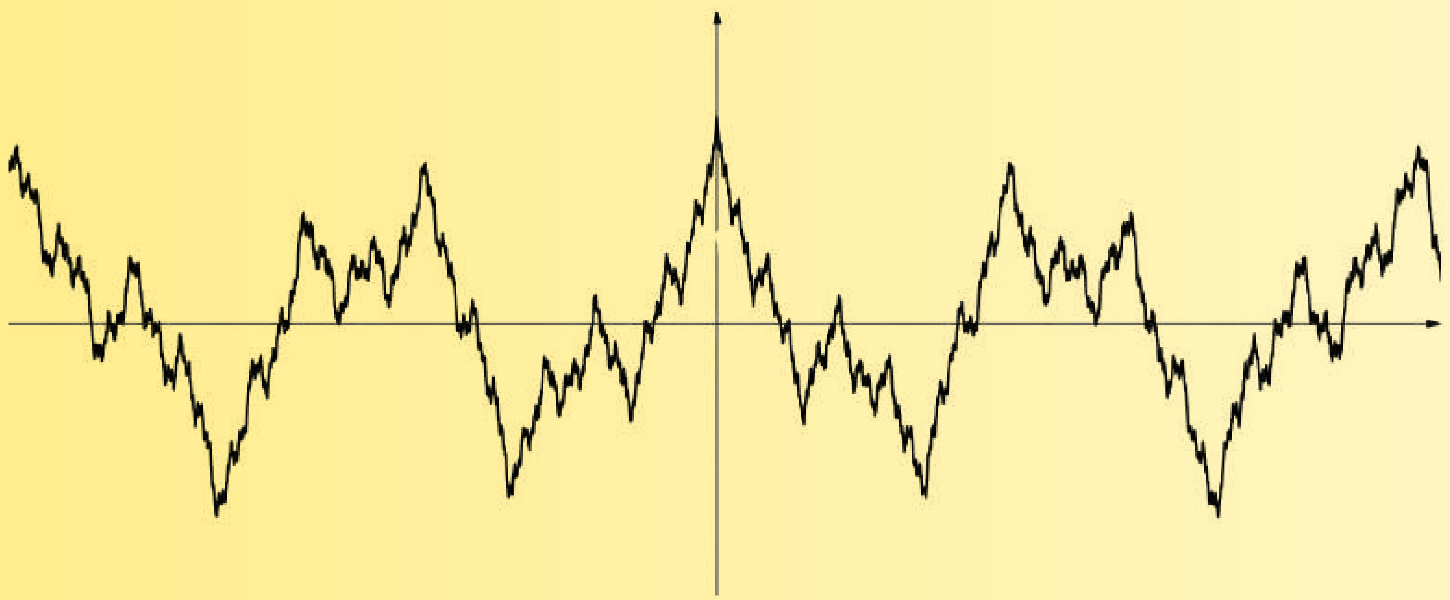
Fourier series

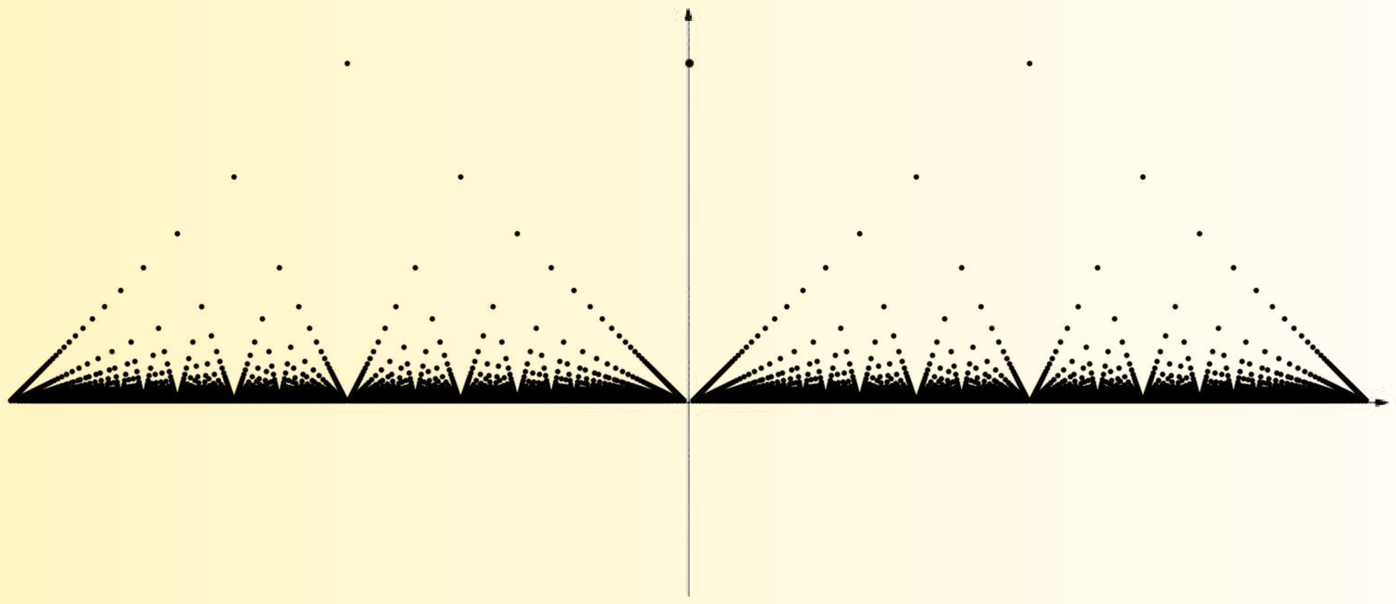
Coefficients do not converge to zero

Series converges on a dense subset of $\mathbb{R}$



Analytic monsters...





$$x \mapsto \sum_{n=0}^{\infty} a^n \cos(b^n \pi x)$$

$$x \mapsto \begin{cases} 1, & \text{if } x = 0 \\ \frac{1}{q}, & \text{if } x = \frac{p}{q} \text{ with } \gcd(p,q) = 1 \\ 0, & \text{if } x \in \mathbb{R} \setminus \mathbb{Q} \end{cases}$$

Weirstrass's function (1872)

Continuous but nowhere differentiable

Thomae's function (1875)

Discontinuous on $\mathbb{Q}$ but Riemann integrable

1747

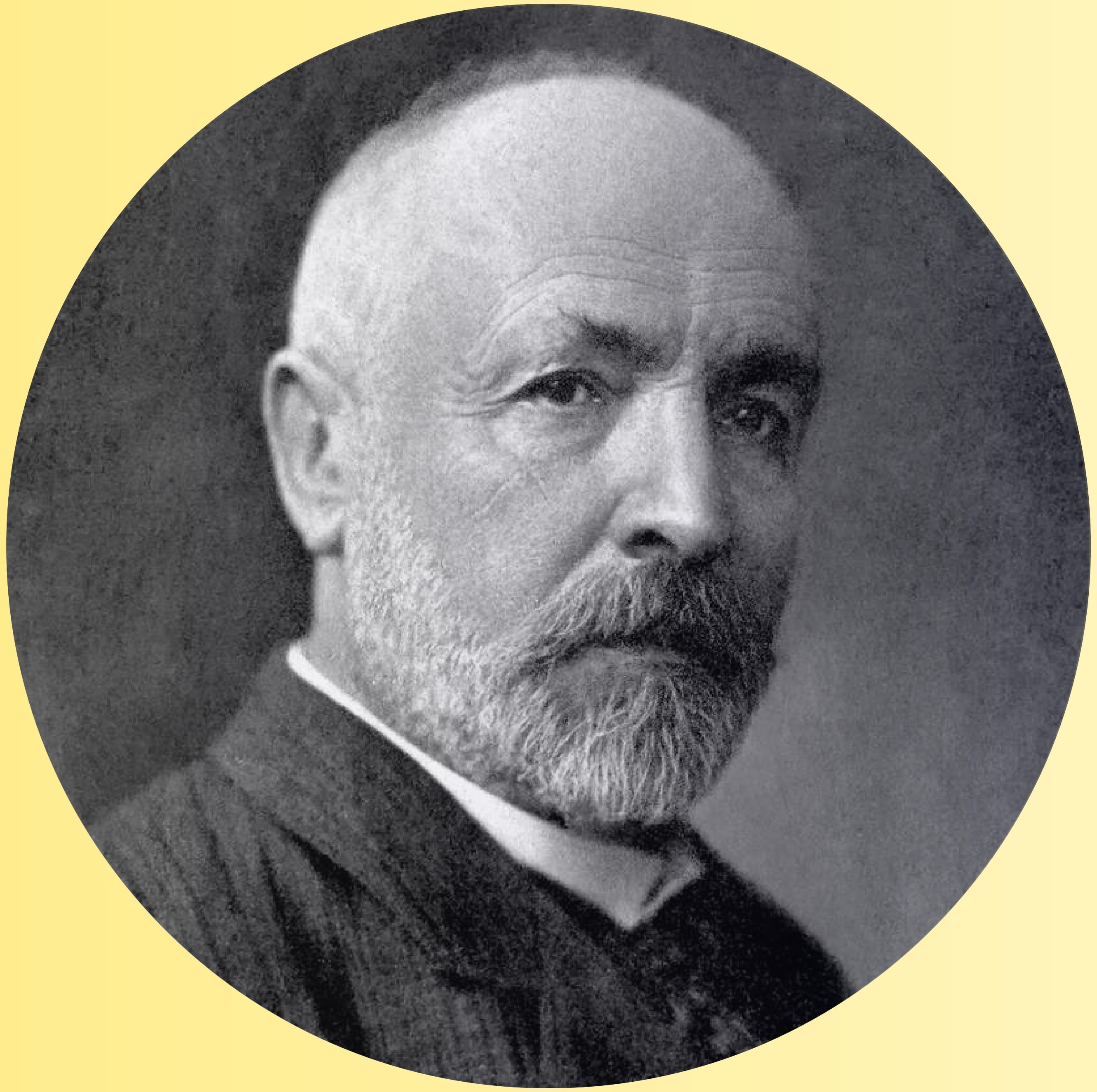
1800

1869

1900

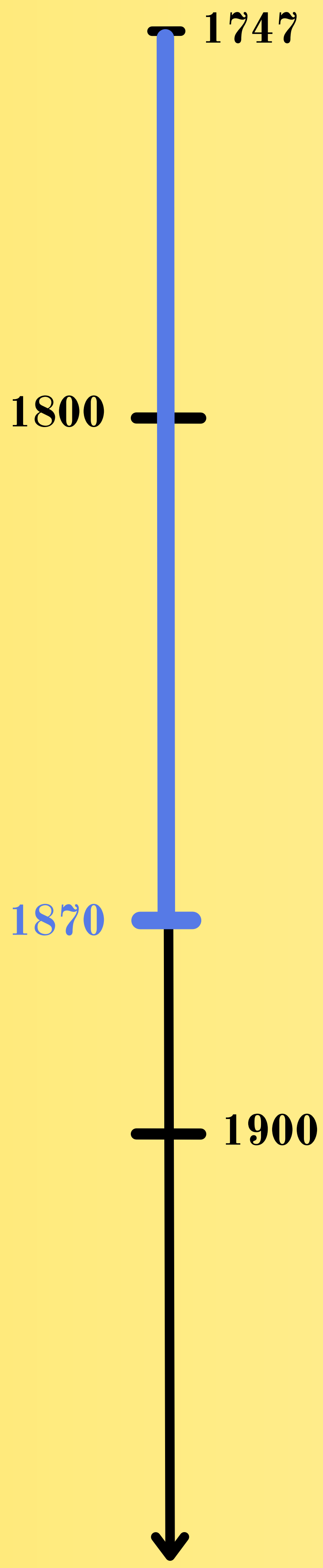
Cantor’s study of sets

1869



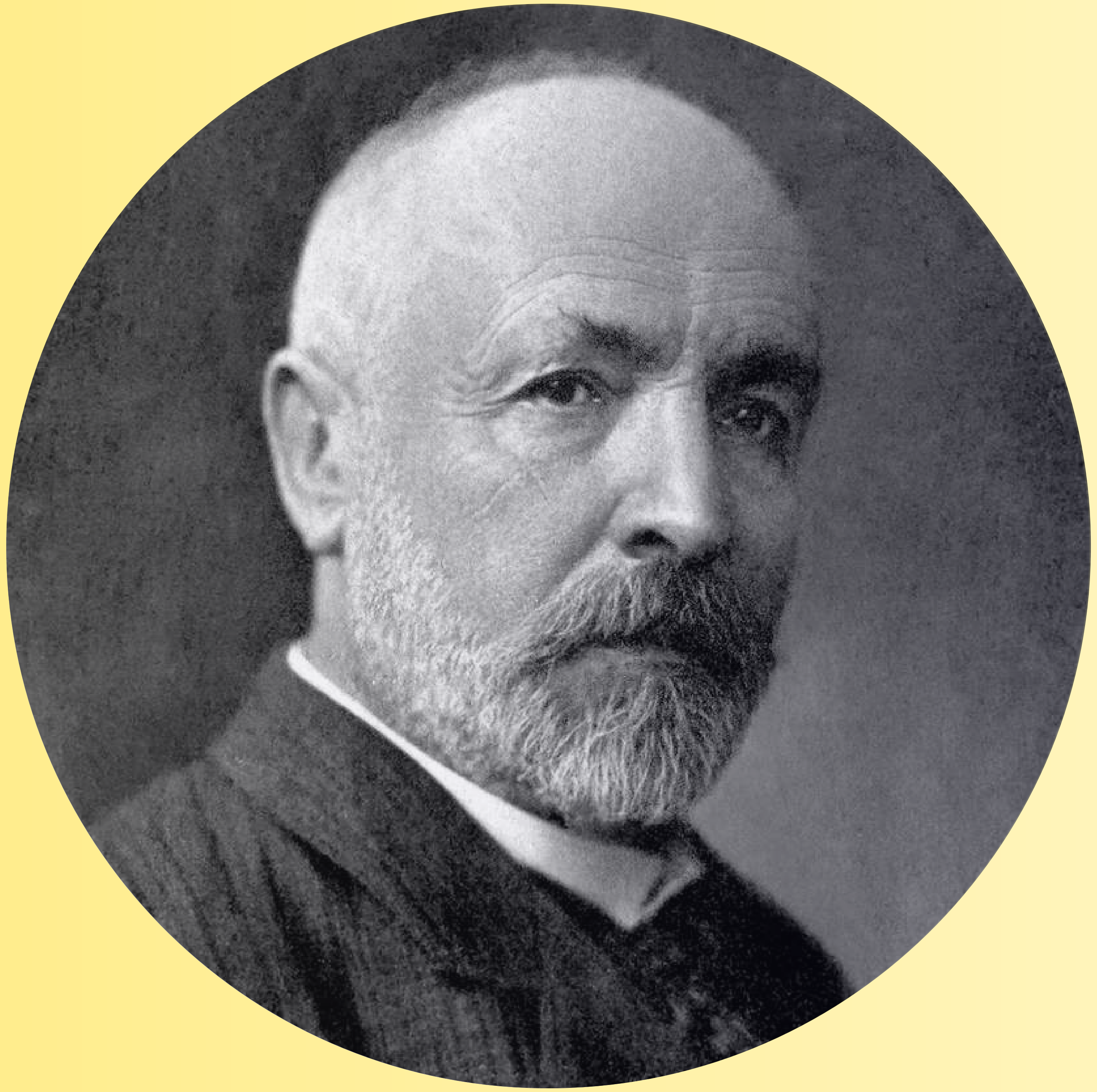
Georg Ferdinand Ludwig Philipp Cantor
[1845 - 1918]

Heinrich Eduard Heine
[1821 - 1881]



Cantor's study of sets

The following year ...
1870



Georg Ferdinand Ludwig Philipp
Cantor
[1845 - 1918]

Cantor's Unicity Theorem (First Edition)

"If an equation is of the form

$$0 = C_0 + C_1 + C_2 + \ldots + C_n + \ldots$$

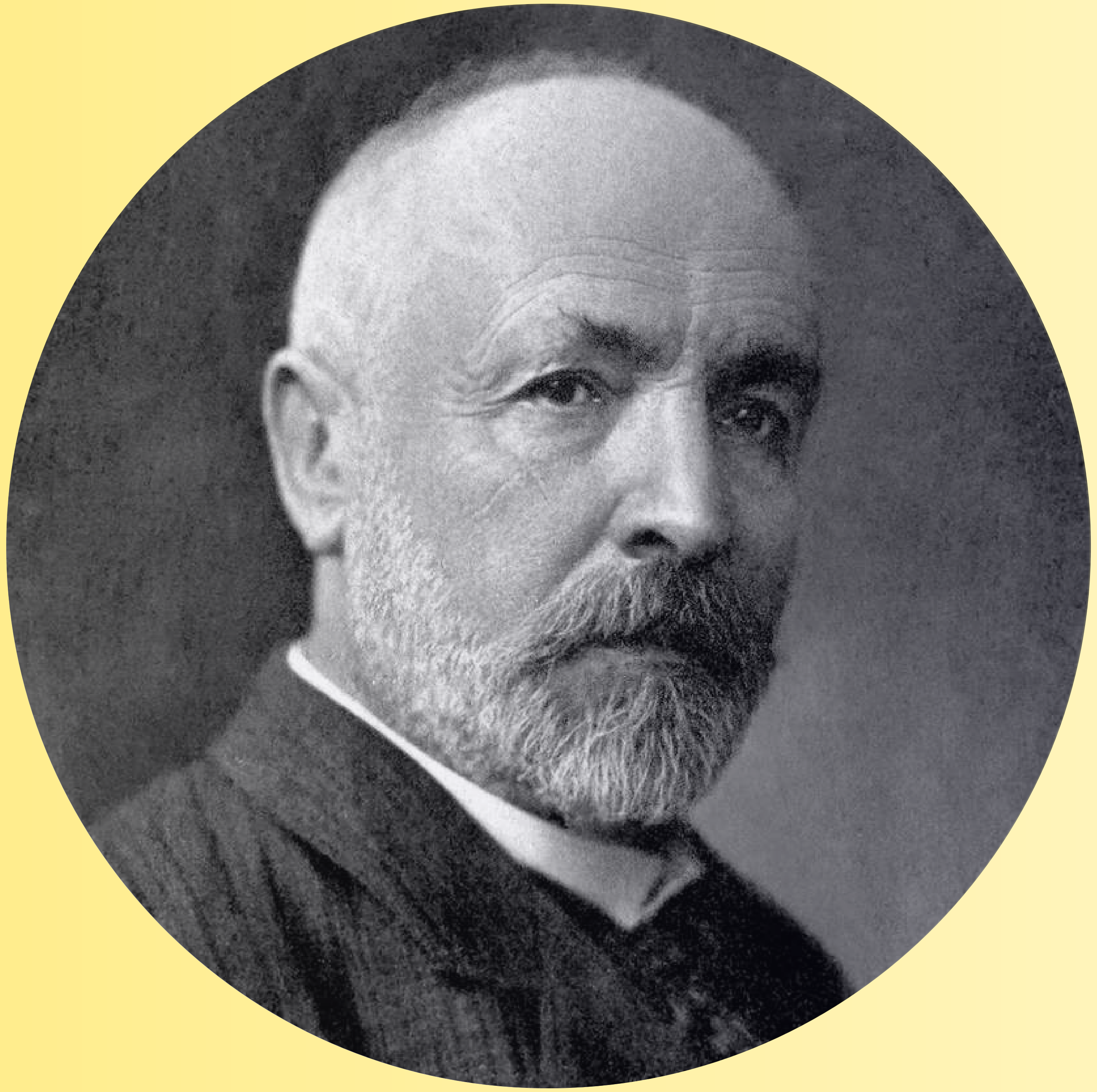
where $C_0 = \frac{1}{2} d_0$ and

$$C_n = c_n \sin(nx) + d_n cos(nx),$$

holds for all values of x in $[0, 2\pi]$ *, I say that we will have* $d_0 = 0, c_n = d_n = 0$ *."*

Cantor's study of sets

The following year ...
1871



Georg Ferdinand Ludwig Philipp Cantor
[1845 - 1918]

Cantor's Unicity Theorem (Second Edition)

"If an equation is of the form

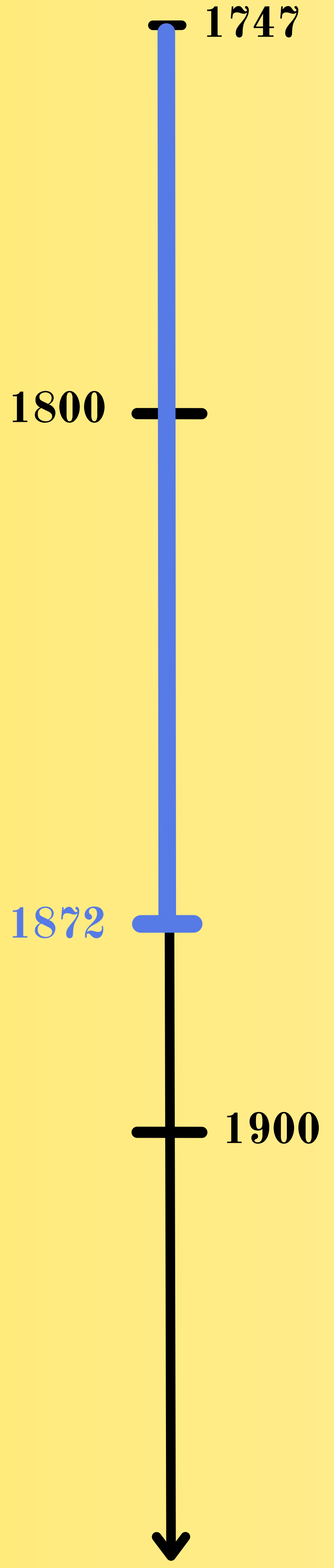
$$0 = C_0 + C_1 + C_2 + \ldots + C_n + \ldots$$

where $C_0 = \frac{1}{2} d_0$ and

$$C_n = c_n \sin(nx) + d_n cos(nx)$$

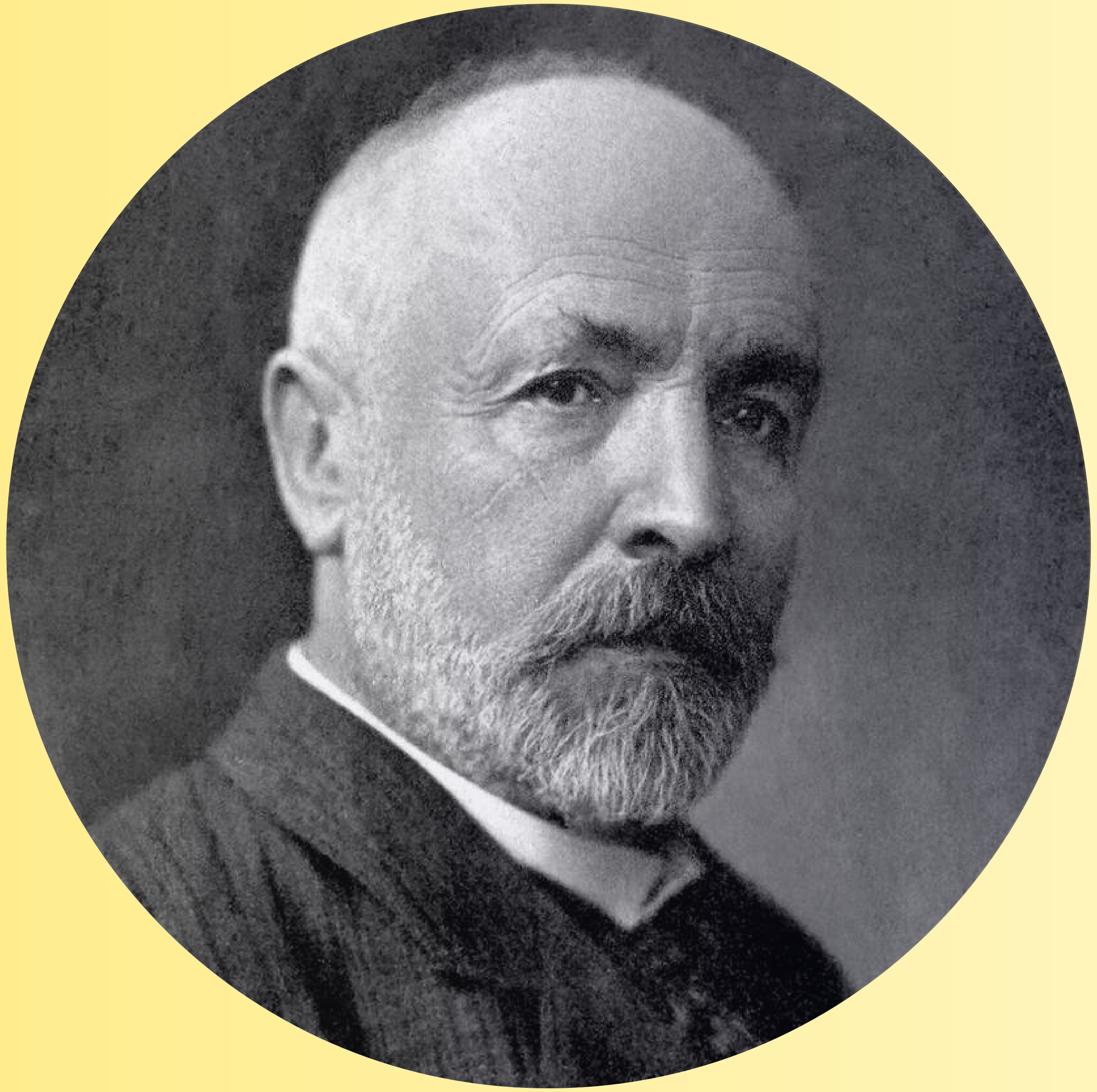
holds for all values of x in $[0, 2\pi]$, *except on finitely many ones I say that we will have*

$$d_0 = 0, c_n = d_n = 0 \ ."$$



Cantor’s study of sets

The following year ...
1872



Georg Ferdinand Ludwig Philipp Cantor
[1845 - 1918]

Creation of the Real Numbers from the Rational Numbers

For $x \in \mathbb{R}$:

$$x \approx \{(a_n) \in \mathbb{Q}^{\mathbb{N}} : a_n \to x\}$$

1747

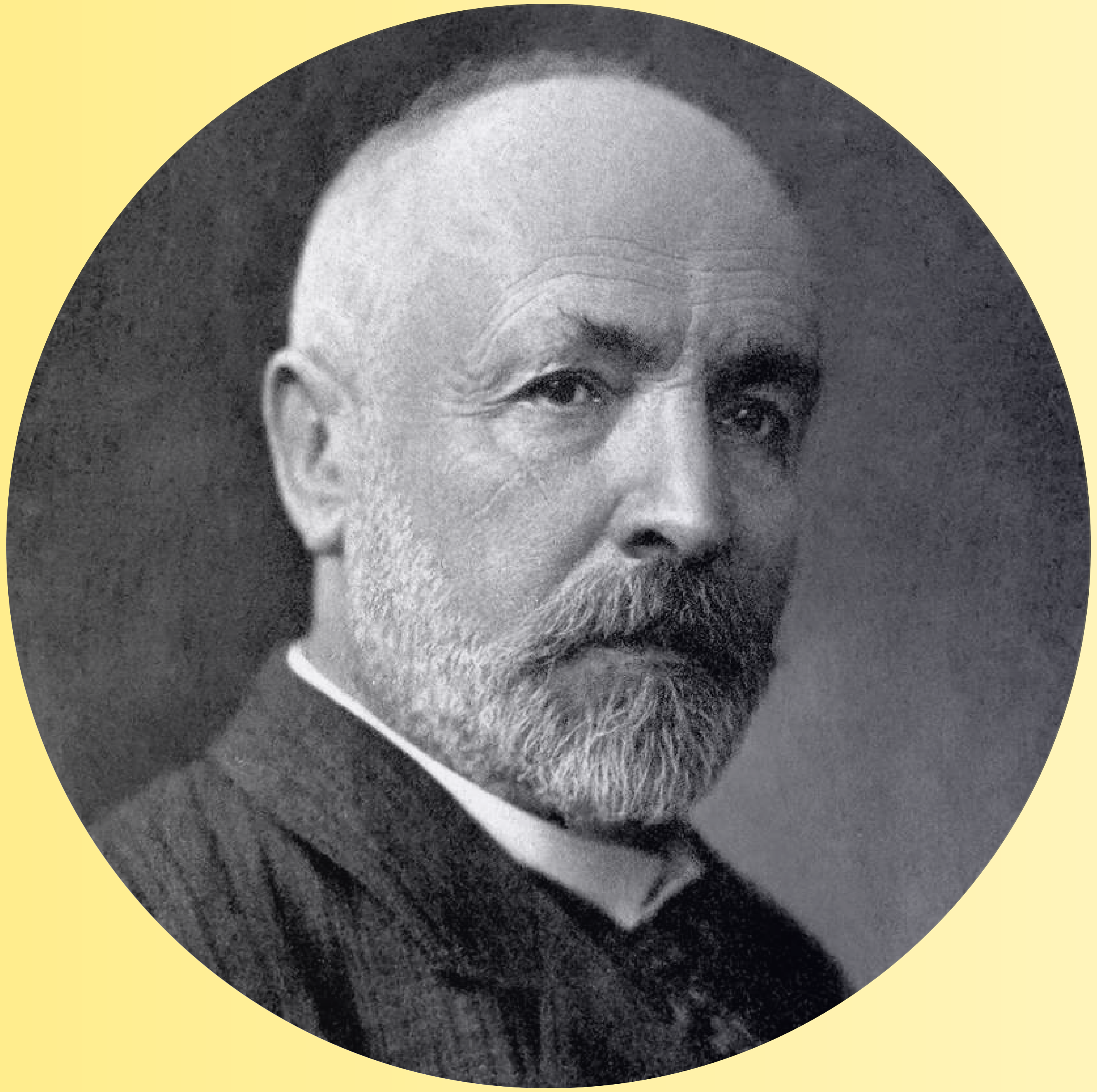
1800

1872

1900

Cantor’s study of sets

1872



Georg Ferdinand Ludwig Philipp Cantor
[1845 - 1918]

Neighborhoods

“I call neighborhood of a point any interval in which this point is contained”

1747

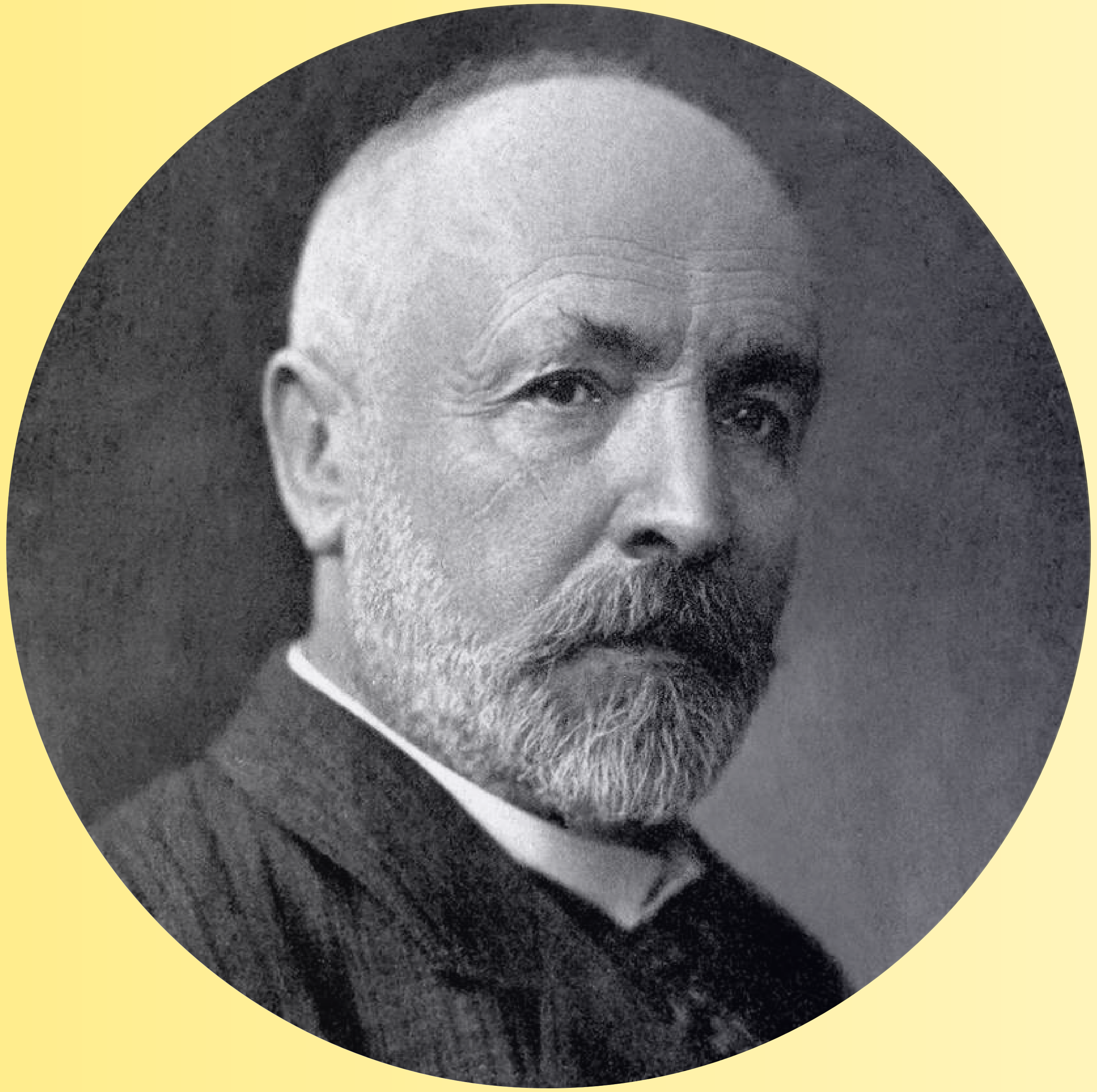
1800

1872

1900

Cantor's study of sets

1872



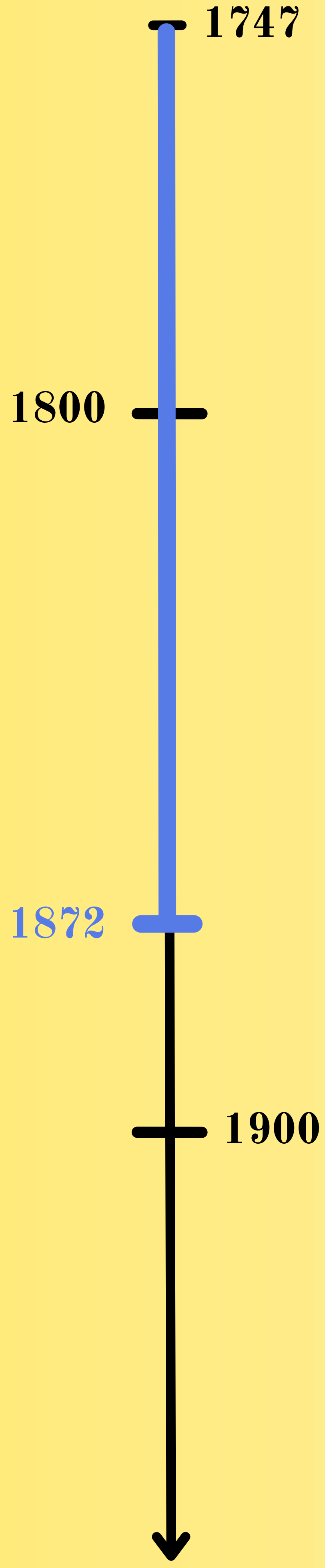
Georg Ferdinand Ludwig Philipp Cantor
[1845 - 1918]

Neighborhoods

"I call neighborhood of a point any interval in which this point is contained"

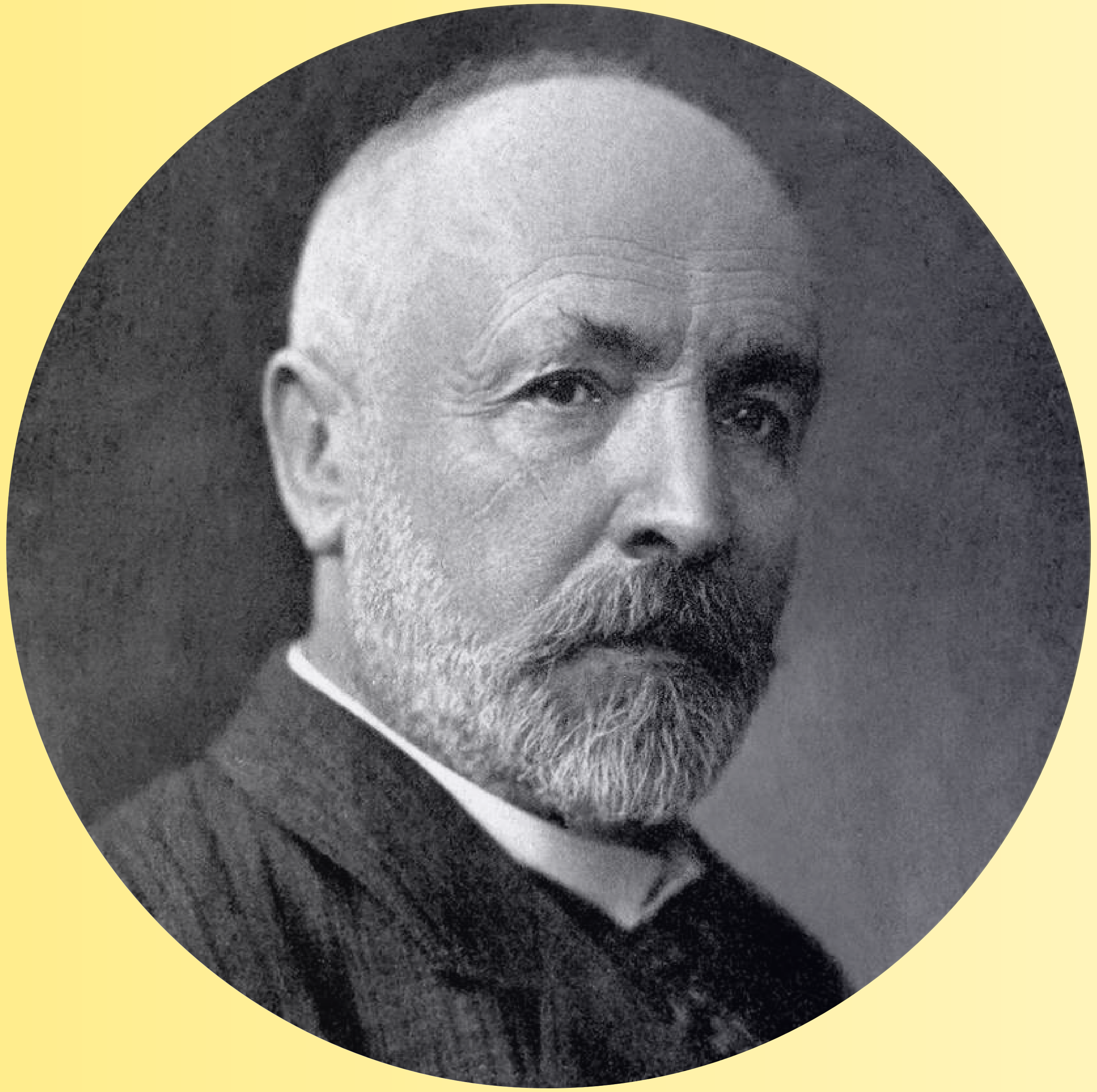
Limit Points

"By limit point of a point system P, I mean a point of the line such that in his neighborhood, there is infinitely many points of the system P."



Cantor's study of sets

1872



Georg Ferdinand Ludwig Philipp Cantor
[1845 - 1918]

Neighborhoods

"I call neighborhood of a point any interval in which this point is contained"

Limit Points

"By limit point of a point system P, I mean a point of the line such that in his neighborhood, there is infinitely many points of the system P."

Isolated Points

"We call isolated point of P any point that, in P, is not at the same time a limit point of P."

Cantor's study of sets

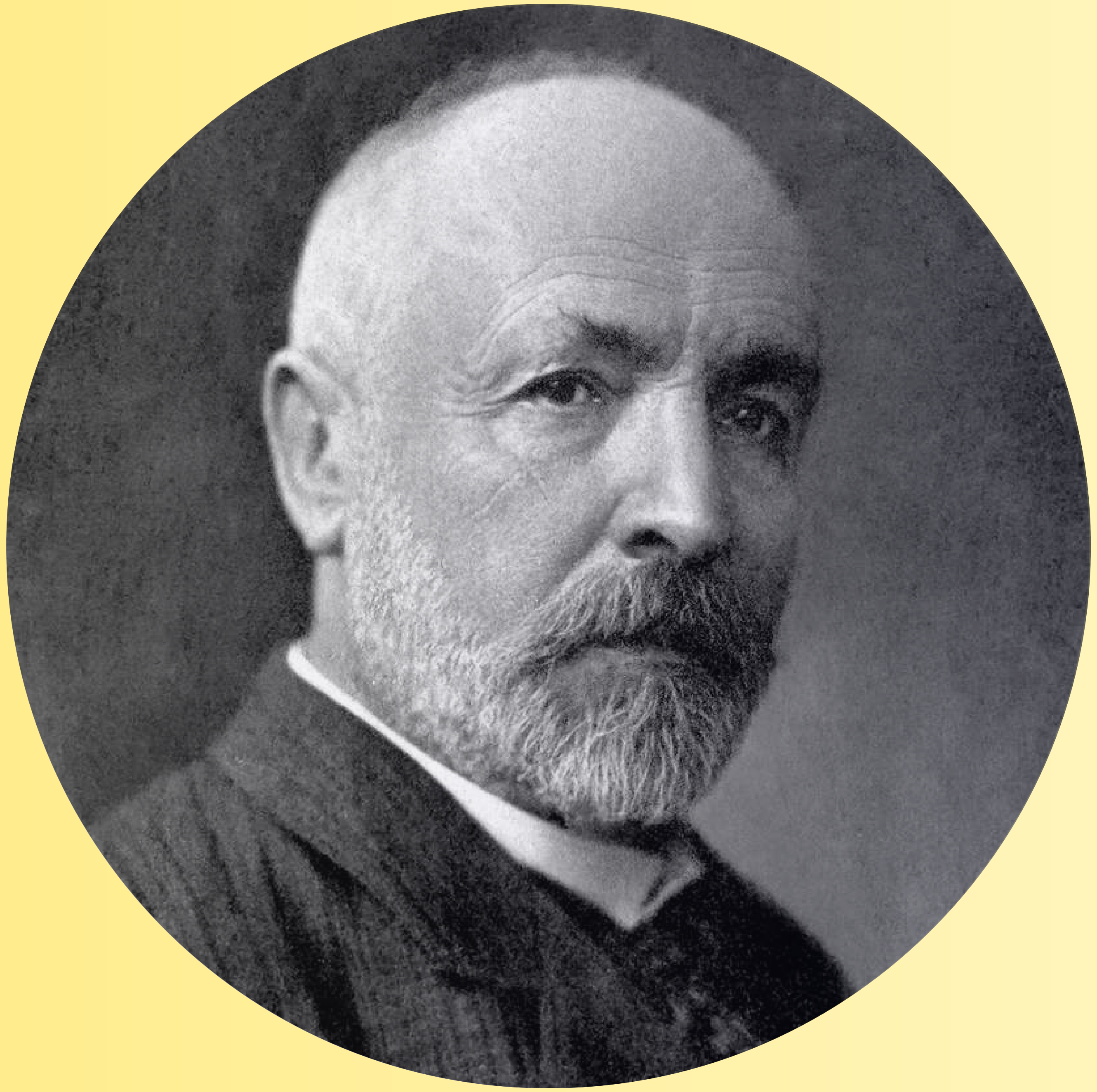
1747

1800

1872

1900

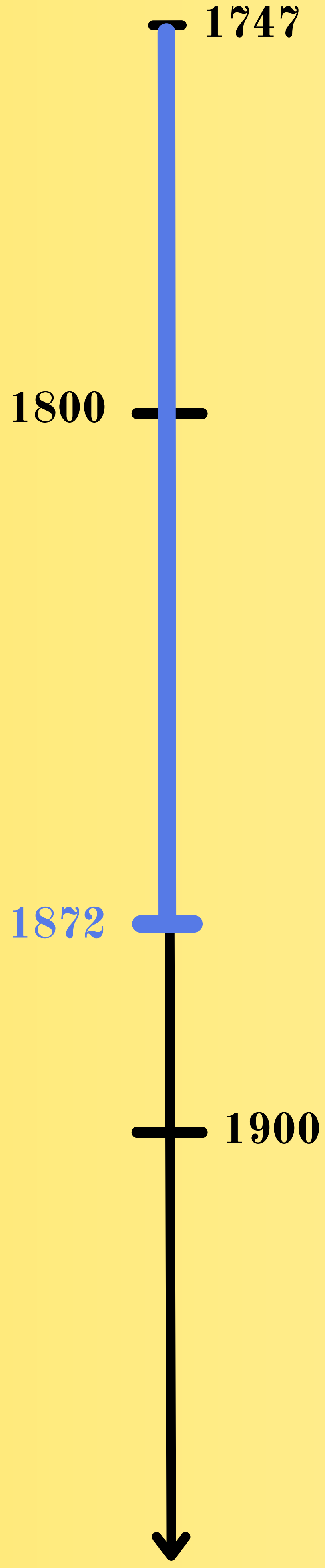
1872



Georg Ferdinand Ludwig Philipp Cantor
[1845 - 1918]

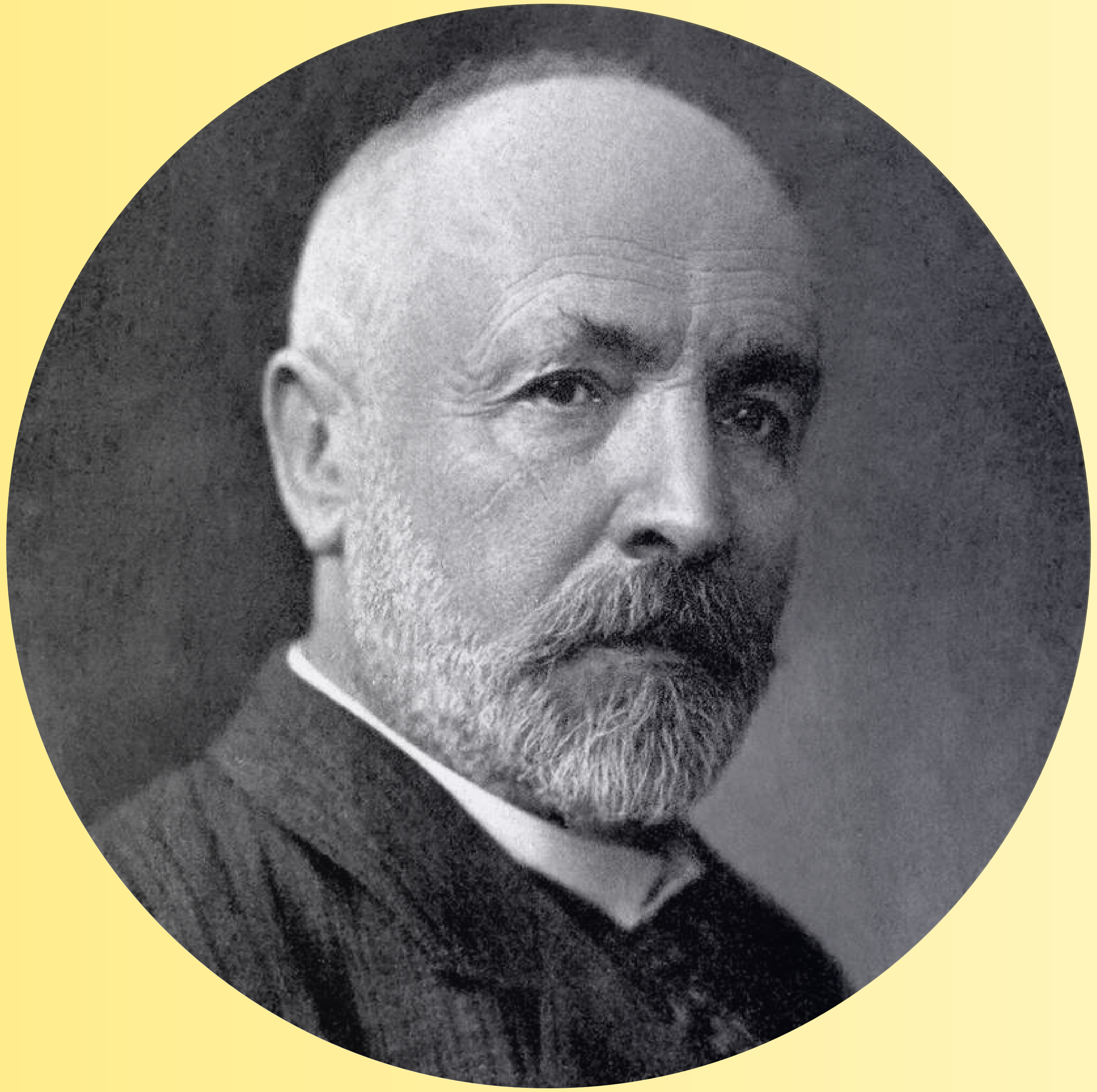
Derived System

The derived system of P, called P', is the system of the limit points of P.



Cantor's study of sets

1872



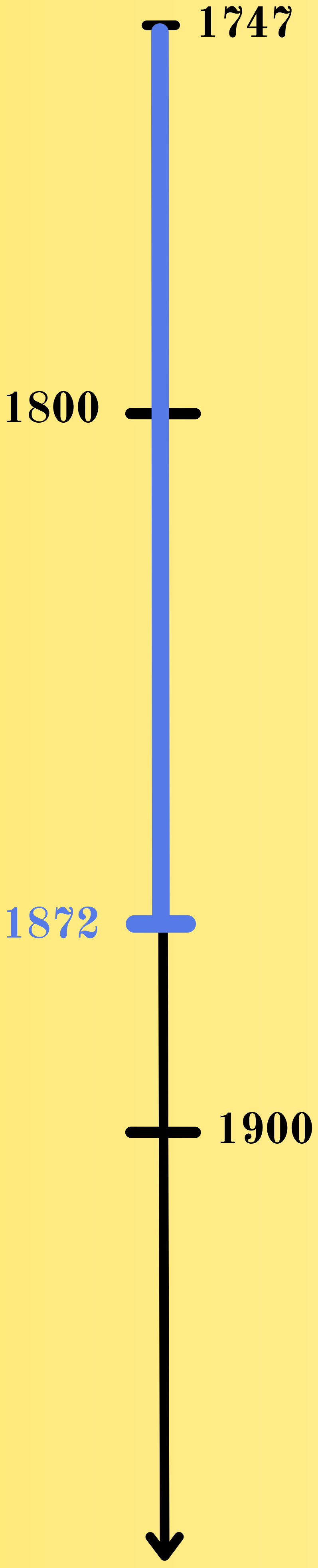
Georg Ferdinand Ludwig Philipp Cantor
[1845 - 1918]

Derived System

The derived system of P, called P', is the system of the limit points of P.

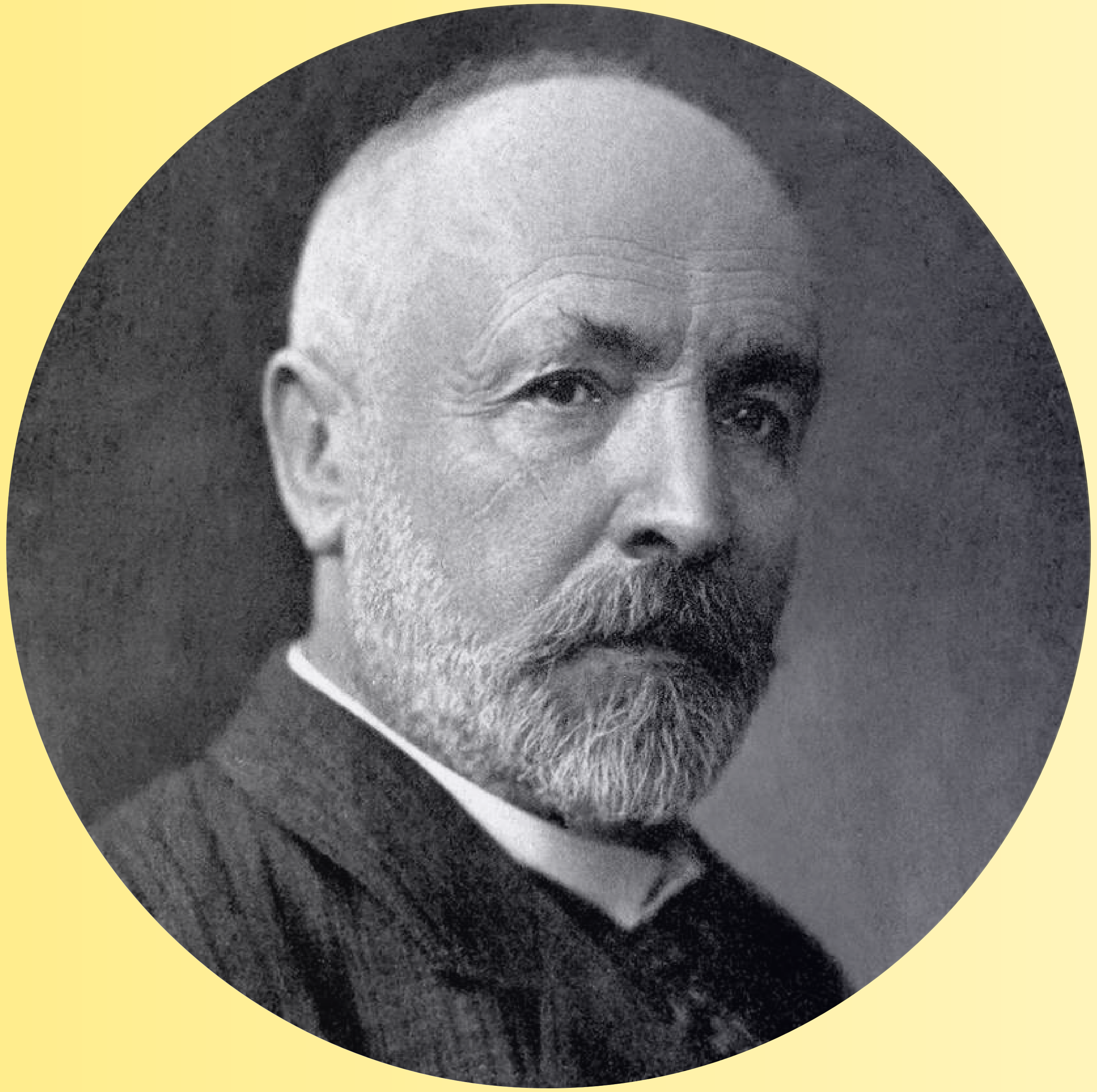
ν^{th} Derived System

By deriving the system P ν times, we get the derived system $P^{(\nu)}$ from P.



Cantor's study of sets

1872



Georg Ferdinand Ludwig Philipp Cantor
[1845 - 1918]

Derived System

The derived system of P, called P', is the system of the limit points of P.

ν^{th} Derived System

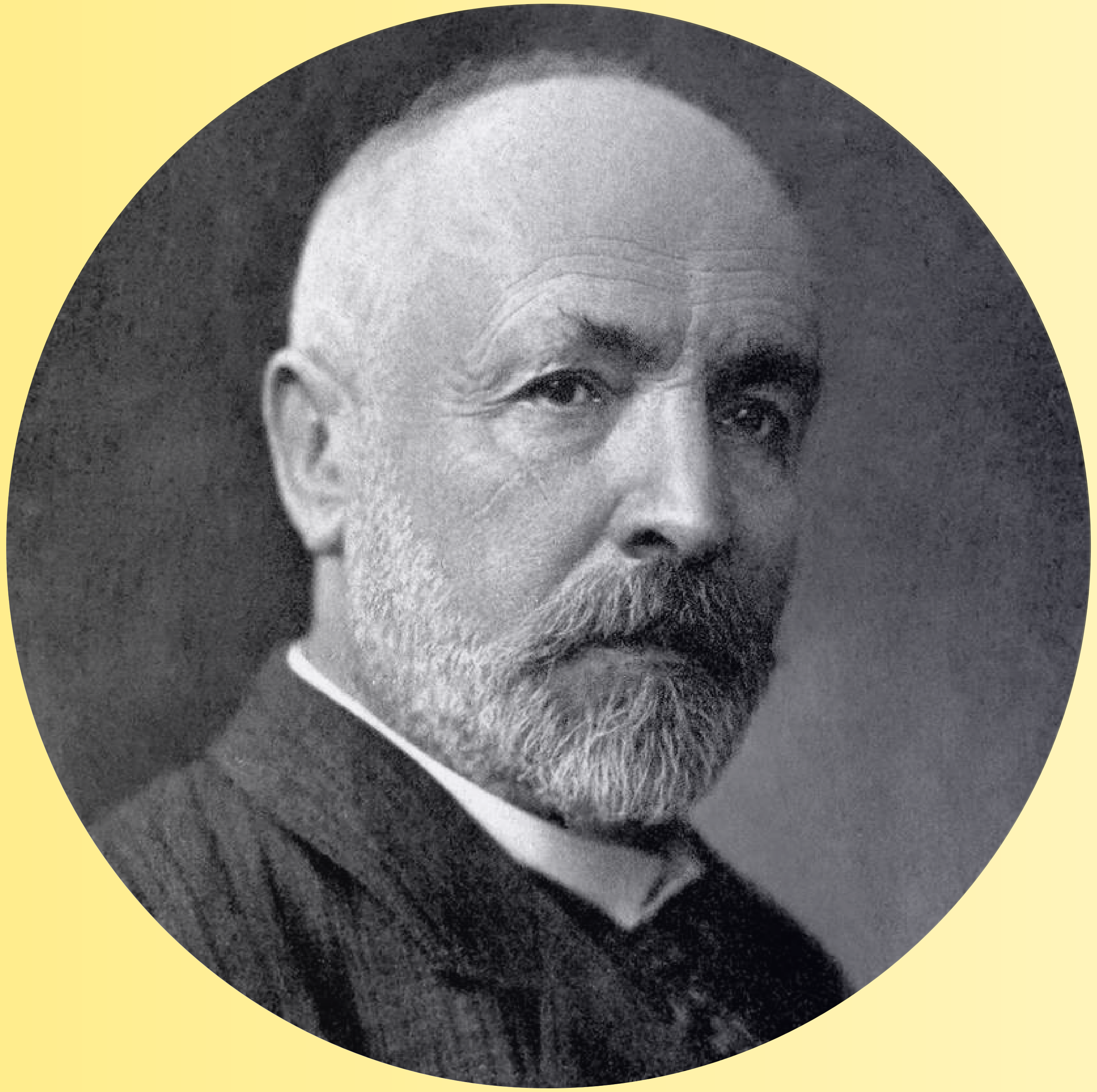
By deriving the system P ν times, we get the derived system $P^{(\nu)}$ from P.

System of the ν^{th} species

A system P is of the ν^{th} species if $P^{(\nu)}$ contains finitely many points.

Cantor's study of sets

1872



Georg Ferdinand Ludwig Philipp Cantor
[1845 - 1918]

Cantor's Unicity Theorem (Final Edition)

"If an equation is of the form

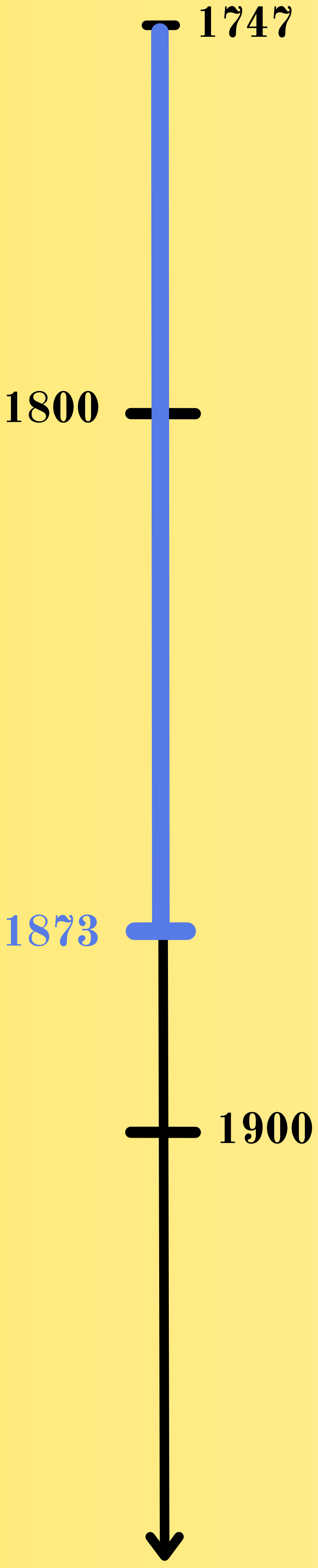
$$0 = C_0 + C_1 + C_2 + \ldots + C_n + \ldots$$

where $C_0 = \frac{1}{2} d_0$ and

$$C_n = c_n \sin(nx) + d_n cos(nx)$$

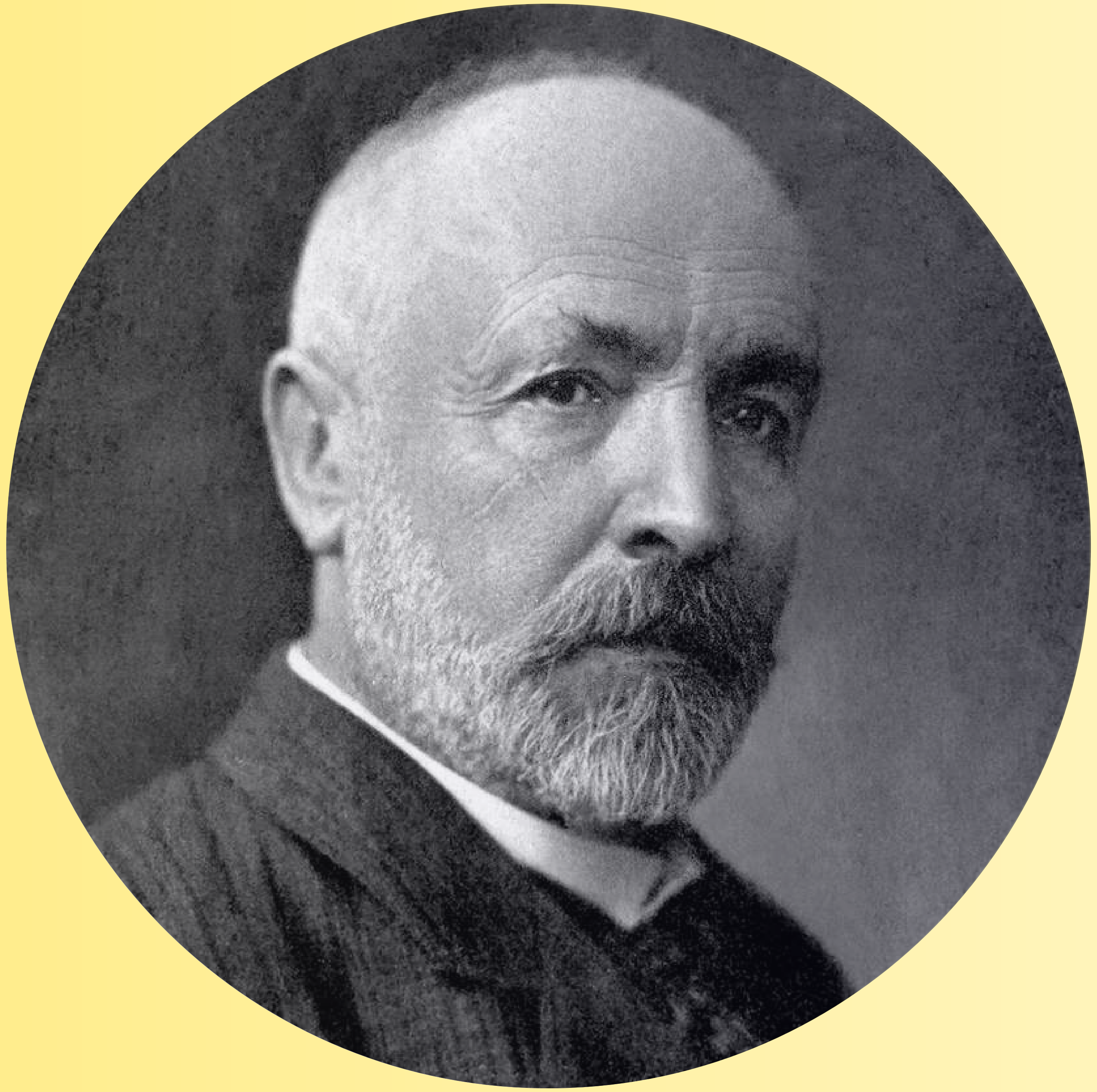
holds for all values of x in $[0, 2\pi]$ *, except on a set P of the v-th species where v is a whole number as large as we want, I say that we will have*

$$d_0 = 0, c_n = d_n = 0$$ *."*



Cantor’s study of sets

The following year...
1873



Georg Ferdinand Ludwig Philipp Cantor
[1845 - 1918]

Julius Wilhelm Richard Dedekind
[1831 - 1916]

1747

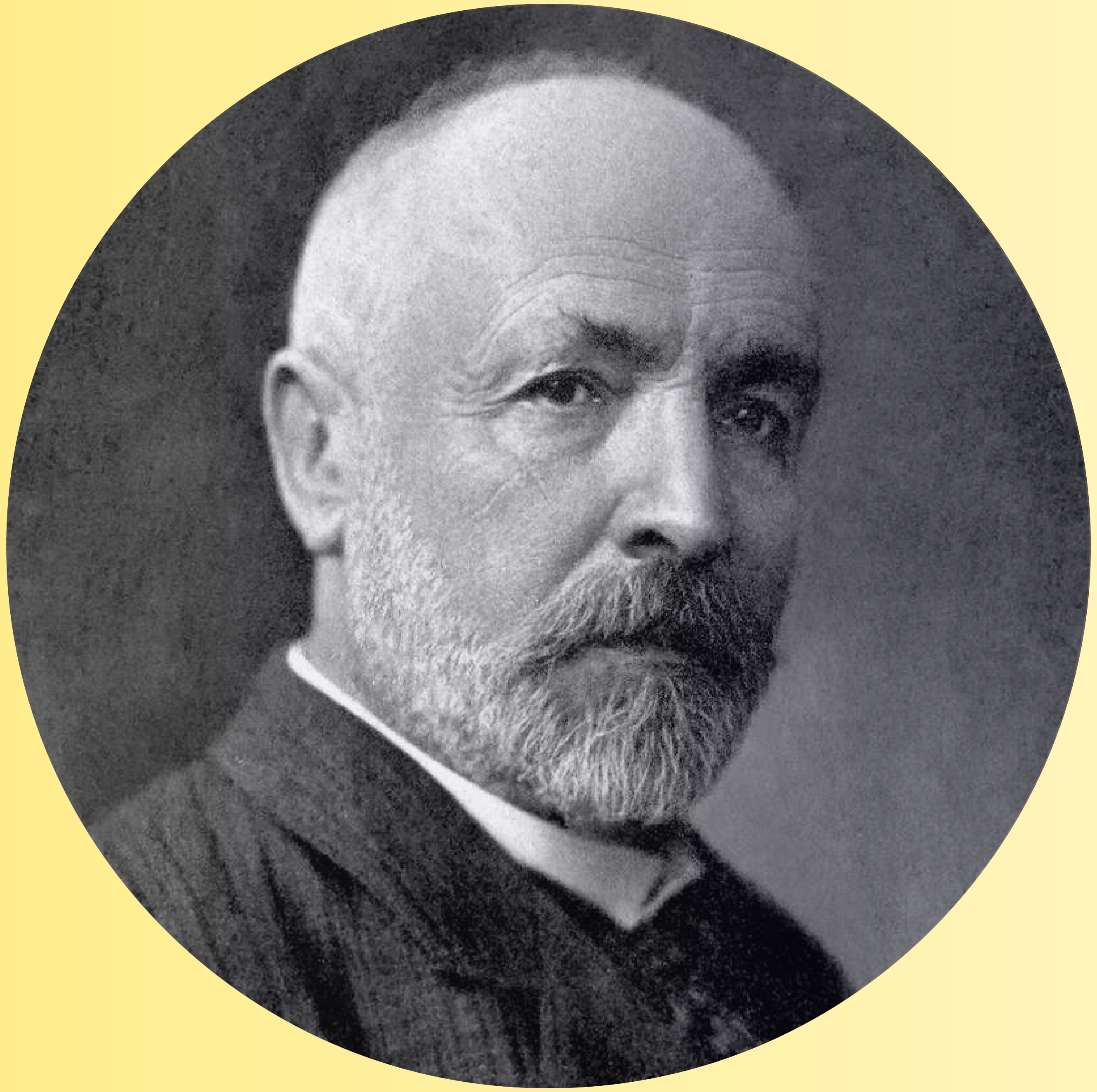
1800

1874

1900

Cantor's study of sets

The following year...
1874



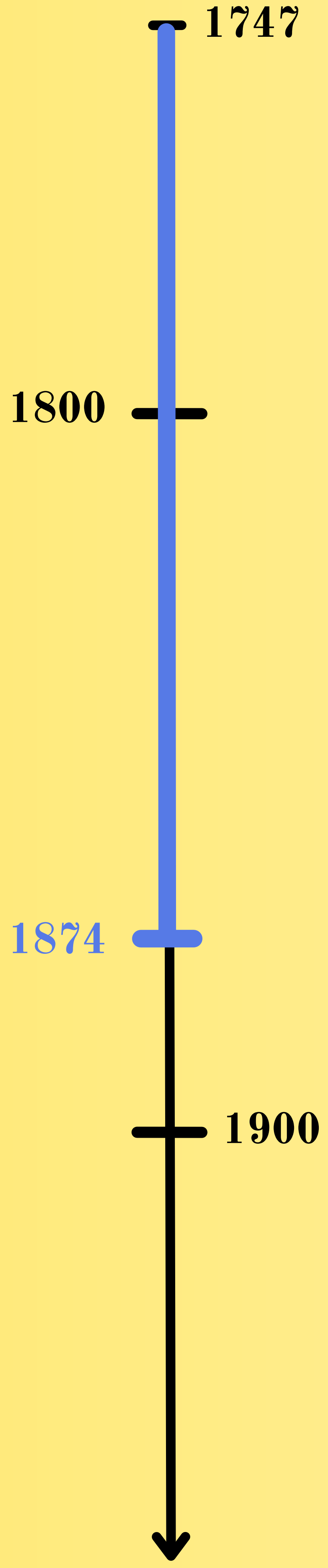
Georg Ferdinand Ludwig Philipp Cantor
[1845 - 1918]

Cantor's First Power Theorem

"Given a sequence

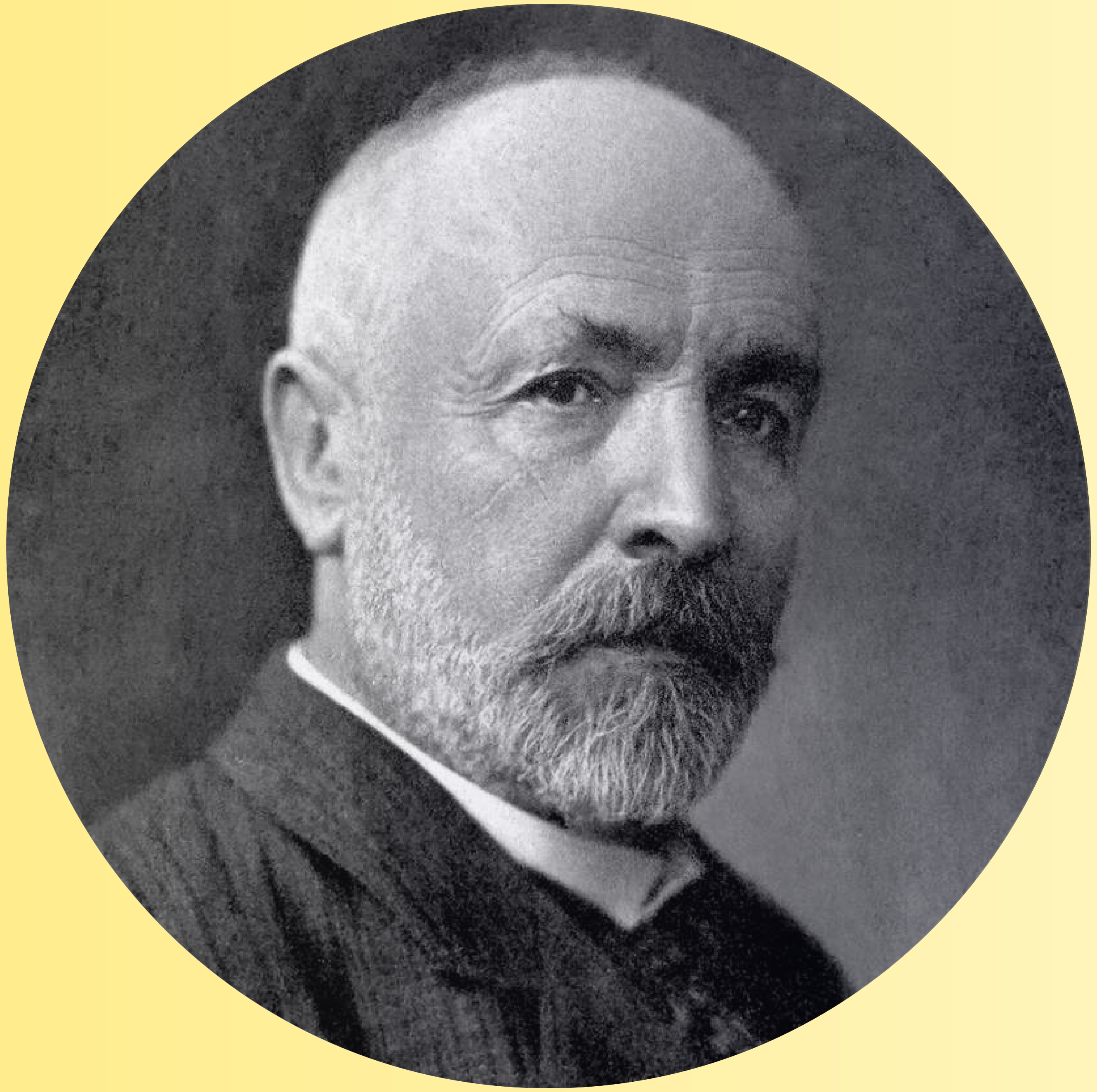
$$u_1,\ u_2,\ u_3,\ \ldots$$

of distinct real numbers determined by arbitrary law, we can find in each interval $(\alpha,\ \beta)$ *a number* ν *that is not contained in the sequence"*



Cantor's study of sets

The following year...
1874



Georg Ferdinand Ludwig Philipp Cantor
[1845 - 1918]

Cantor's First Power Theorem

"Given a sequence

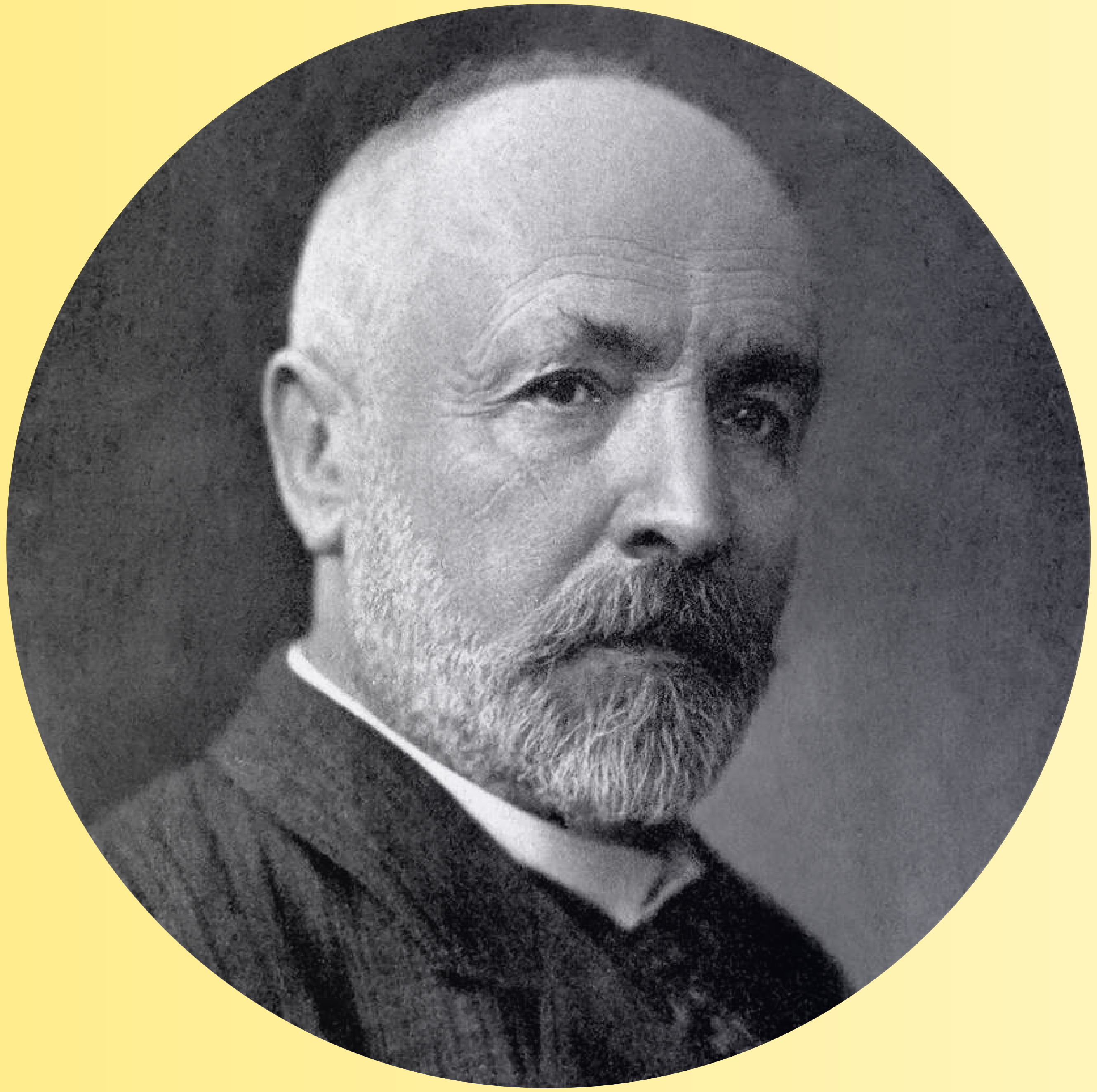
$$u_1,\ u_2,\ u_3,\ \ldots$$

of distinct real numbers determined by arbitrary law, we can find in each interval $(\alpha,\ \beta)$ *a number* ν *that is not contained in the sequence"*

$$\mathbb{N} \neq \mathbb{R}$$

Cantor's study of sets

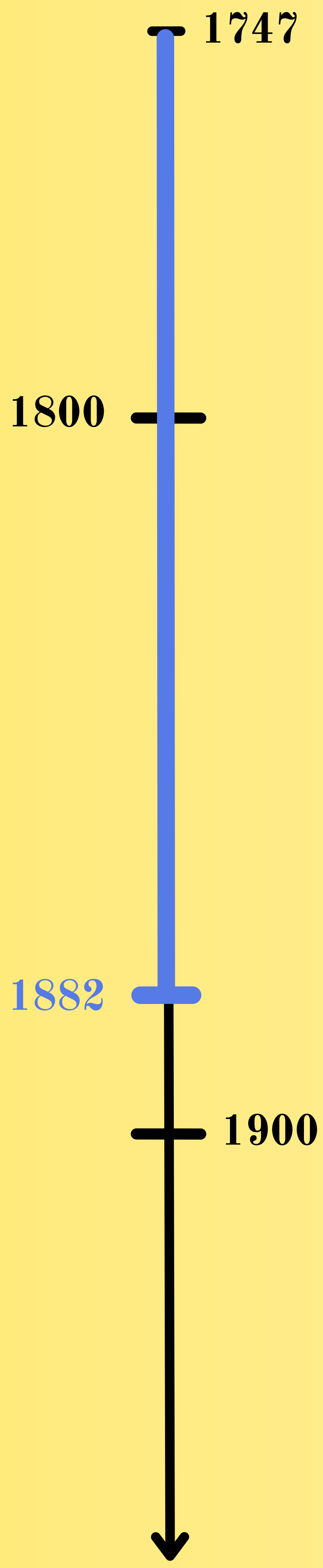
3 years later...
1877



Georg Ferdinand Ludwig Philipp
Cantor
[1845 - 1918]

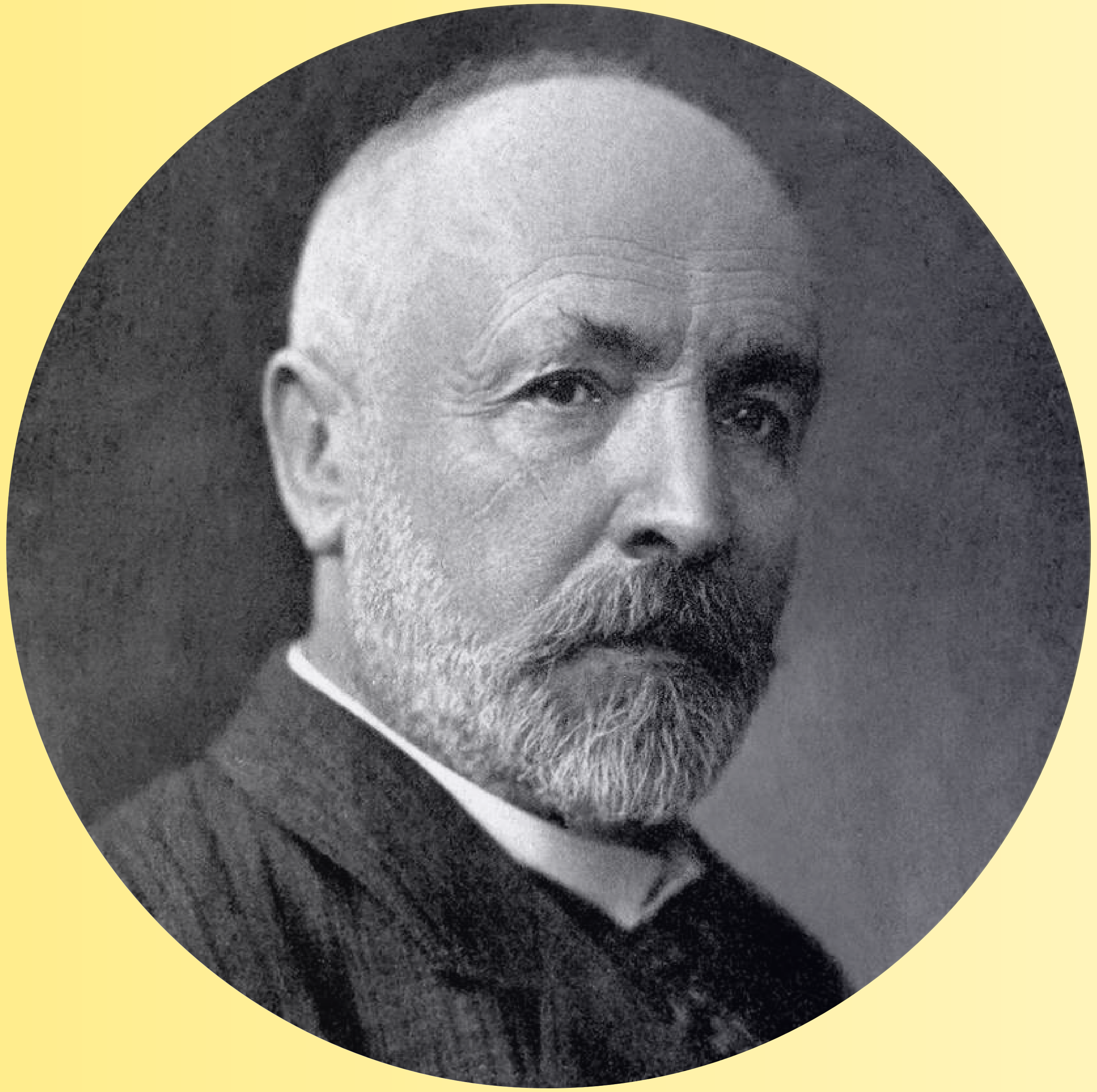
Cantor's Second Power Theorem

$$\mathbb{R} \equiv \mathbb{R}^2 \equiv \mathbb{R}^3$$



Cantor's study of sets

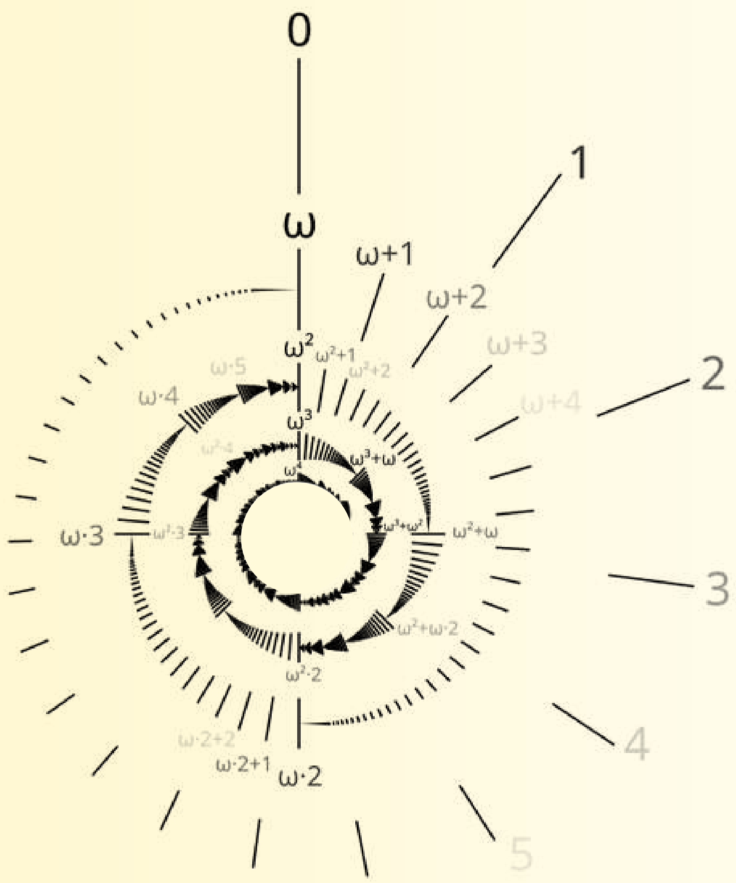
1882



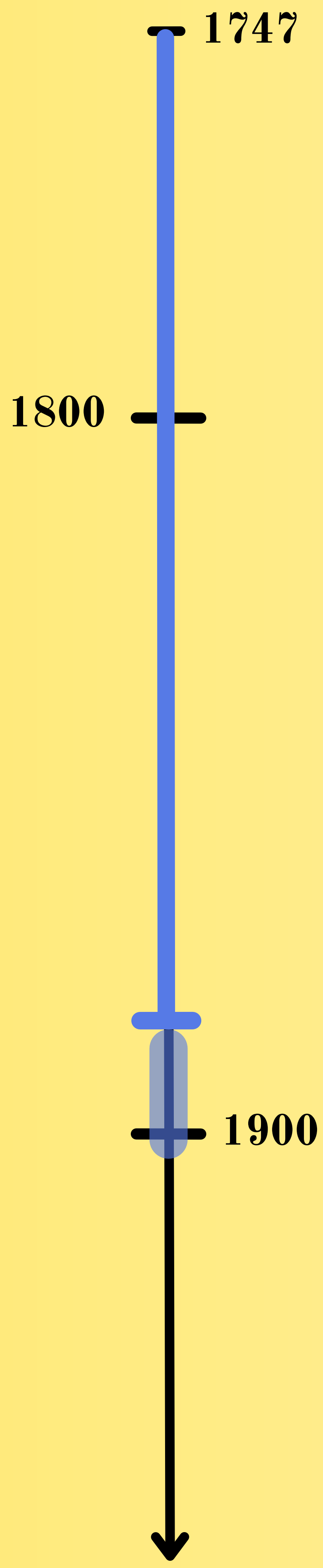
Georg Ferdinand Ludwig Philipp Cantor
[1845 - 1918]

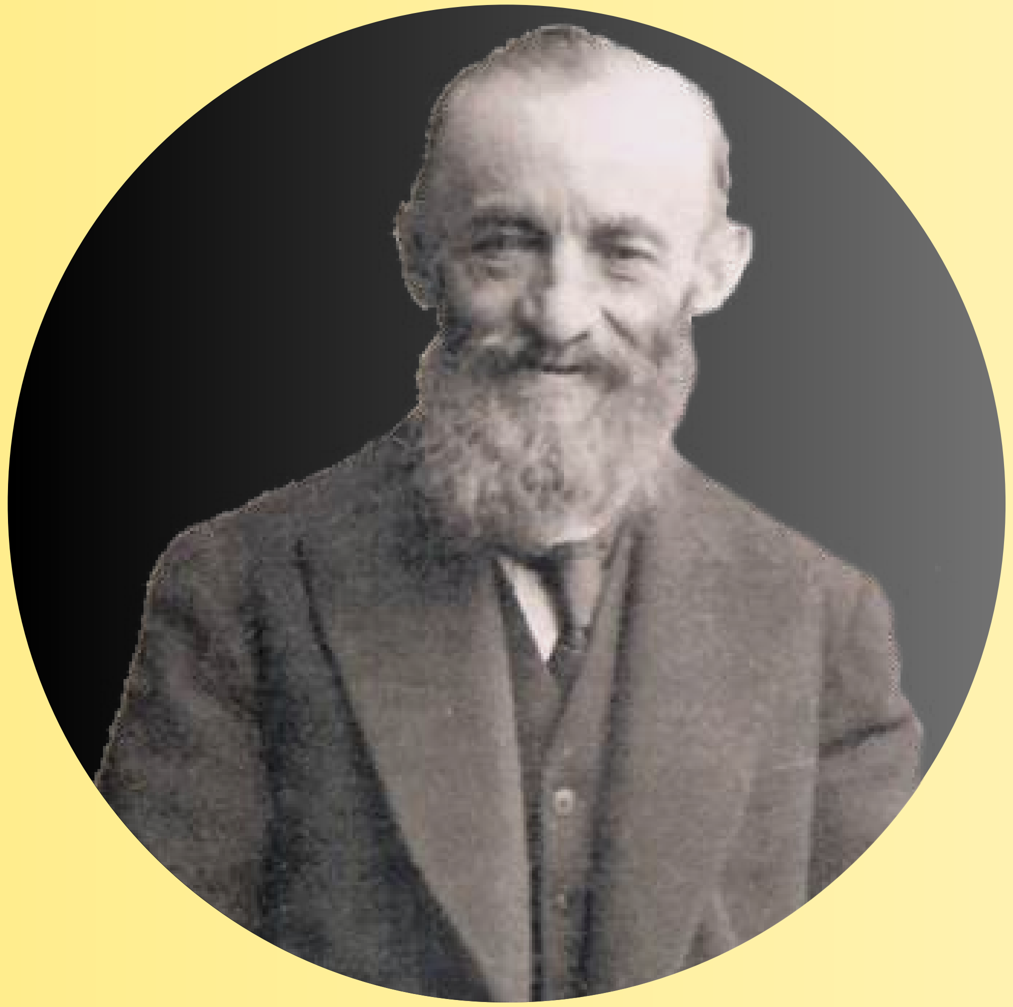
FONDEMENTS D'UNE THÉORIE GÉNÉRALE
DES ENSEMBLES
PAR
G. CANTOR
à HALLE a. S.

FONDATIONS OF A GENERAL SET THEORY
By G. CANTOR



From Lebesgue to now





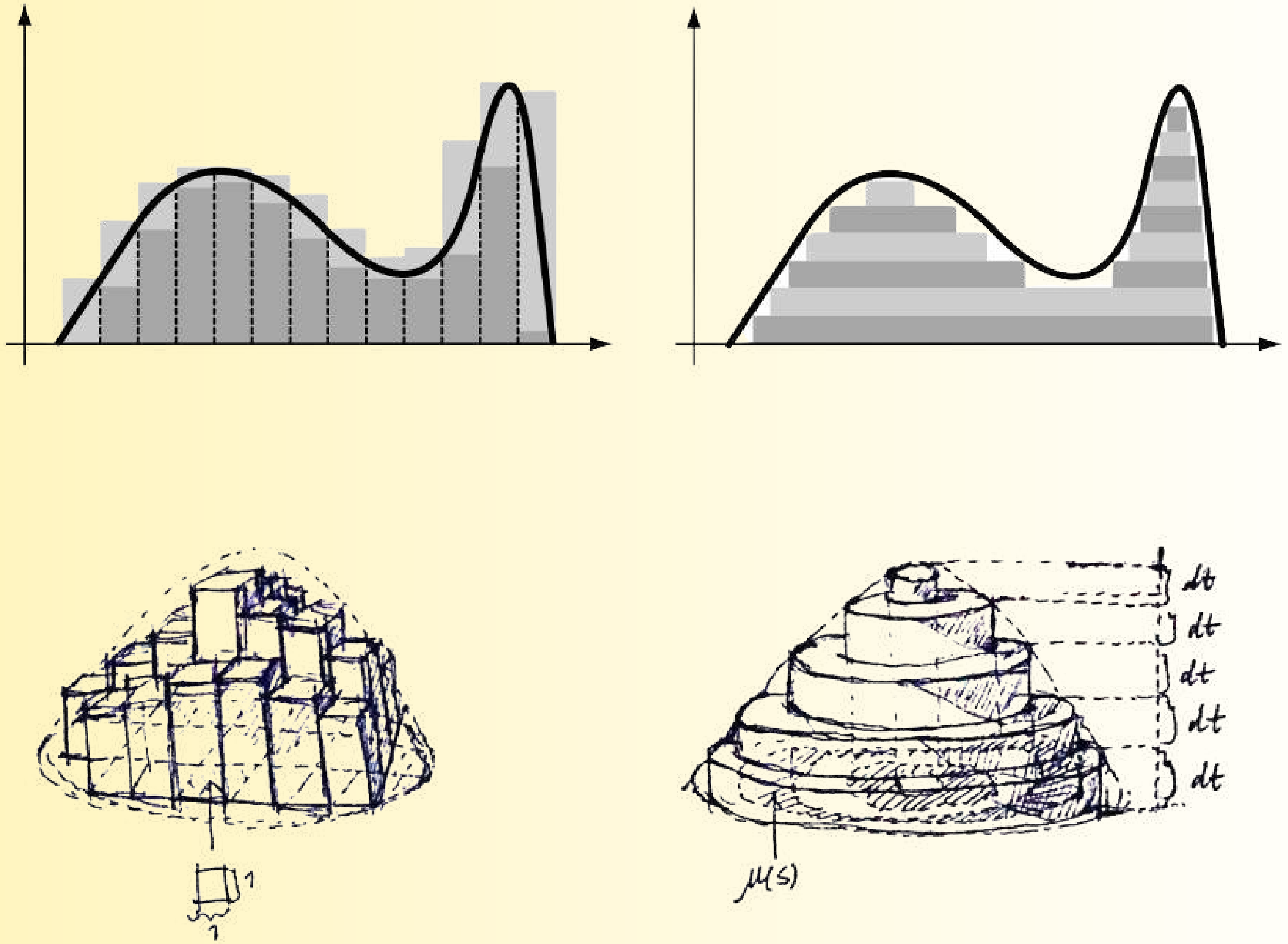
Guiseppe Peano
[1858 - 1932]

Camille Jordan
[1838 - 1922]

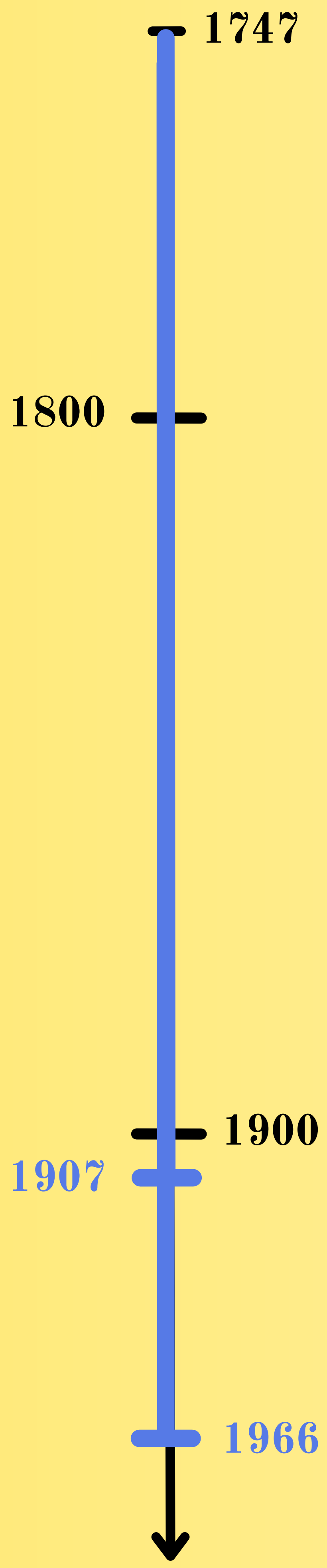
Emile Borel
[1871 - 1956]

From Lebesgue to now

Henri Léon Lebesgue
[1875 - 1941]



Conceptual difference between Riemann's Integral and Lebesgue's Integral



From Lebesgue to now

Frigyes Riesz
[1880 - 1956]

Ernst Sigismund Fischer
[1875 - 1954]

Lennart Axel Edvard Carleson
[1928 - alive and well)

Riesz-Fischer Theorem (1907)

A function has a convergent Fourier Series in the sense of L^2 if and only if it is in L^2.

Carleson's Theorem (1966)

If a function is in L^2, then its Fourier Series converges almost everywhere.

The End